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# A separable programming model incorporating linear demand functions for grains and vegetable oils: an analysis of United States agriculture in 1985

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A SEPARABLE PROGRAMMING MODEL INCORPORATING LINEAR DEMAND FUNCTIONS  
FOR GRAINS AND VEGETABLE OILS: AN ANALYSIS OF UNITED STATES AGRICULTURE  
IN 1985

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A separable programming model incorporating linear demand  
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United States agriculture in 1985

by

Shashanka Bhide

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## CHAPTER I. INTRODUCTION

American agriculture is faced with a number of issues, some of which have been persistent problems for many years and some of which have their origin in the recent past. The low and gyrating farm incomes and farm prices relative to nonfarm incomes and prices have been persistent problems in U.S. agriculture (52). More recently, changing nature of agricultural production and management have raised concern among farmers and policymakers regarding the viability of the family farm. Certain aspects of agricultural production have attracted growing public concern regarding environmental quality and conservation of natural resources. Specifically, use of certain agricultural chemicals and run-off from feed-lots cause pollution of water streams. Intensive use of heavy machinery for field operations in farming leads to greater soil loss due to wind erosion. In recent years, the energy situation characterized by rising prices and predictions of reduced availability of fossil fuels has been affecting agriculture in numerous ways.

The present study is an attempt to analyze the impact of alternative energy price and supply situations on U.S. agriculture. Hence, although the many issues facing agriculture are important and complex, only those related to the "energy issue" in general are discussed in some

detail below.

### The Energy Debate

Beginning with the formation of Organization of Petroleum Exporting Countries (OPEC) in 1973, prices of all energy sources have increased dramatically. The price changes for different energy sources are illustrated in Figure 1.1. Studies such as those by the Central Intelligence Agency (65) have led to a growing perception of possibilities of fossil fuel shortages in near future. President Carter's national energy plan (20) states, "the diagnosis of the U.S. energy crisis is quite simple: demand for energy is increasing, while supplies of oil and natural gas are diminishing", and also notes, "the principal oil-exporting countries will not be able to satisfy all the increases in demand expected to occur in the U.S. and other countries throughout the 1980's". The importance of energy situation in economic and political, or in national and international contexts cannot be overstated. The crisis in the energy sector has a bearing on a number of problems, some of which are real and some potential:

1. the balance of payment problem for many countries around the world;
2. inflation in the economies around the world and prospects of recession;



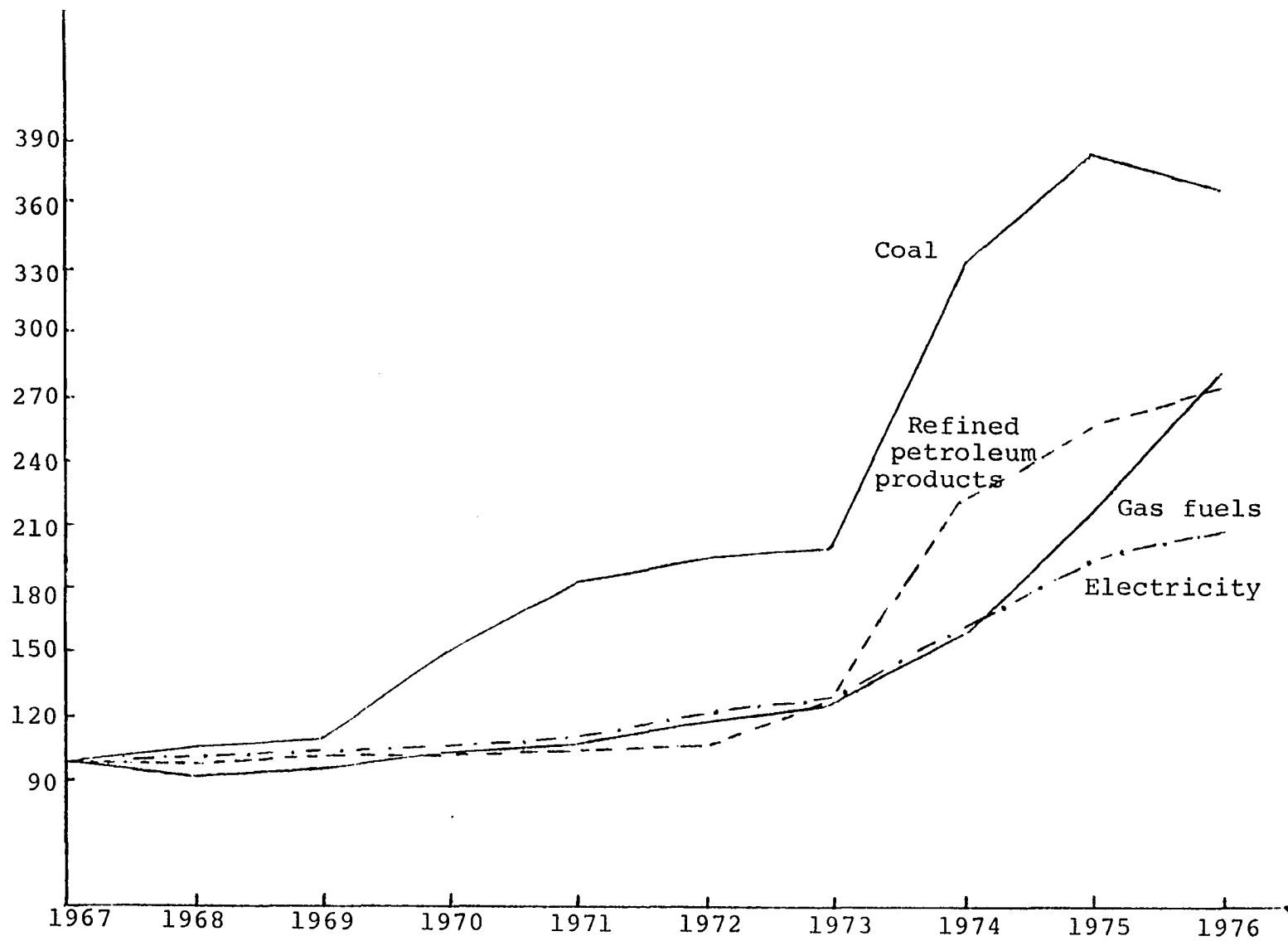


Figure 1.1. Index of energy prices (1967=100) (70)

3. economic development of nonoil-producing developing countries;
4. the world food crisis; and
5. social and political problems in the oil-producing, developing nations.

In the United States, the "energy crisis" and its effects have led to a national debate towards developing a national energy policy. Despite the confused state of the debate (53) and lack of a concrete energy policy, certain elements of the evolving policy are clearly emerging. Provision of greater economic incentives in and encouragement for,

1. development of alternative (replenishable) sources of energy;
2. expansion of existing domestic energy sources;
3. conservation and more efficient use of energy; and
4. a switch by the users from scarce to relatively abundant energy sources,

appear to be the central points of the evolving national energy policy (20, 42).

There is a general realization that energy will continue to be an expensive input. As energy is an essential input in all sectors of the economy, changes in energy prices and energy supply will have significant effects on the output of the economy. Studies (4, 21) have been conducted on the impact of changes in energy

situations on different sectors of the economy in the United States. In the following section a brief review of agriculture vis-a-vis the energy situation is presented.

### Energy and Agriculture

The food system in the United States has developed into a system characterized by intensive use of energy in farm production, processing and transportation of farm products (54). According to Stout (57), of the total energy consumed in the United States, 16.5 percent is accounted for by the food system. The components of the food system are: a) farm production, b) food processing, c) transportation, d) wholesale and retail trade, and e) home preparation and the consumer. Food processing accounts for 33 percent of the energy consumed in the food system. Agricultural production accounts for 18 percent and home preparation 30 percent. Transportation and trade account for 3 and 6 percent respectively. The above break down of energy consumption within the food system is given by Hirst (30).

In production agriculture, agricultural chemicals require the largest energy input followed by machinery, transportation, irrigation, and livestock production (57). Stout (57) also notes gasoline and diesel are the major sources of energy used in production agriculture, followed

by natural gas and liquid petroleum gas (LPG).

The above description of the relationship between energy and agriculture implies that reduction in energy use in agricultural production alone does not significantly affect the national energy balance. But, another facet of the issue, importance of energy to agriculture, presents a different picture.

All of the items in agricultural production except for human labor require for use and manufacture fossil fuels, or power from other sources such as hydro-electric power, and petroleum-based products. Input items such as fertilizers, herbicides and pesticides have increased significantly in use during the period from 1945 onwards. Use of labor directly on farm reduced to less than a one-third during the period from 1940 to 1970 and is continuing to decline. The changes in resource structure of American agriculture are illustrated in Figure 1.2. These changes have evolved over time in response to technological advancement and economic considerations in farm production (15). Mechanization has replaced much of human and animal labor with machine labor in agricultural production. Fertilizers and agricultural chemicals, along with the use of improved plant varieties have improved the productivity of farmlands. Although the United States uses large amounts of



Figure 1.2. Changes in resource structure of U.S. agriculture: indices of input use (1967=100) (66, 67)

energy per acre, the U.S. farm output per acre is also large (36).

A feature of livestock production in the United States that attracts criticism from some food specialists (for example, Lappe (35)) is the use of cereal grains to feed livestock. Table 1.1 shows that energy required to produce a unit of protein from livestock is significantly higher than from grains or soybeans directly. But as Reid and White (47) point out, if economic conditions dictate, feeding grain to produce animal protein and energy for human food will decrease. With growing energy-related costs, feeding grains to cattle may become less profitable as compared to increased use of roughages as livestock feed.

Several adjustments may be required in the agricultural sector in response to the changes in energy situation. Here, we review some of the adjustments in the farm production sector. Adjustments in the food processing component of the food system are discussed by David (14). Stout (57) lists the following options for farmers to cope with increasing energy costs and scarcity of energy:

1. reduce energy use;
2. substitute enterprises that require less energy;
3. substitute plentiful for scarce forms of energy;
4. use alternative energy sources;
5. modify farm enterprises to increase efficiency; and

Table 1.1. Cultural and dietary energy expended under hypothetical intensive and extensive systems of production (Reid and White (47))

Food source	Mcal of energy expended per kg of protein produced			
	Intensive production <sup>a</sup>		Extensive production <sup>b</sup>	
	Cultural energy	Dietary energy	Cultural energy	Dietary energy
Milk	15	120	6	130
Beef	54	570	32	690
Pork	67	-	-	-
Corn grain	16	-	-	-
Soybeans	6	-	-	-

<sup>a</sup>"Intensive" means the feeding of diets in which concentrates provide 25, 80, and 100 percent of the digestible energy to dairy cows, cattle in the feedlot (after weaning), and pigs, respectively.

<sup>b</sup>"Extensive" means the feeding of all-forage diets (pasture, haylage, and corn silage) after weaning to both dairy and beef cattle.

6. cease farming.

To the list above we may add:

7. develop and harness the energy producing potential of agriculture.

The specific items of choice available to farmers, under various options cited above are many. Some are already available on a practical basis and others are at various stages of development. The driving force behind the adjustments to be made will be the economic benefit associated with such adjustments.

Reduced use of inputs such as fertilizers and pesticides decrease energy use in agriculture. But such a reduction in inputs also causes loss in the productivity of land. If reductions in cost due to lower rates of fertilizer and pesticide application, are smaller than the reduction in income due to loss in output, energy use on farms may not be realized. Attempts to replace chemical or machinery inputs by human labor to maintain the output level also may not be economically attractive. Berry (7) notes that, "at 1970 prices, 1,000 kilocalories of inputs which are considered to expand the area cultivated or materials handled per worker cost about 1.5 cents, compared with 1,000 kilocalories of labor costing \$3."

Reduced tillage or no-till methods of farming were primarily seen as a way to reduce soil erosion and conserve the soil, until recently. But reduced tillage and no-till



farming also reduce the fuel-energy requirements for tillage operations. For example, Allen et al. (2) note that, "the elimination of one field operation on 25 percent of the nation's 1973 corn and soybean acreage (129 million acres) could have saved about 16 million gallons of diesel fuel equivalent". The USDA (69) has projected that nearly half of the nation's more than 300 million acres of planted cropland could be managed by minimum tillage and no-till by 1990. Berry (7) suggests that reduction in labor cost achieved under reduced tillage as compared to conventional tillage provides much of the incentive for adopting reduced tillage.

In the manufacture of inputs for farming sector, use of alternative sources such as coal in place of natural gas or other energy inputs in short supply may be one possible alternative in reducing demand for energy inputs in short supply. In agricultural production, one important use of solar energy (other than photosynthesis) is in crop drying. Other uses of solar energy, for example in irrigation and in obtaining energy from biomass, are at different stages of research. Use of wind power for generating electricity may also become a feasible energy alternative on farms in some regions.

Gasohol - a combination of alcohol from agricultural products like grain (corn) and sugar crops, and gasoline -

has already appeared in the market as a fuel for internal combustion engines. Use of alcohol as a fuel is becoming economically feasible as fossil fuels are becoming increasingly expensive. For example, during 1978-79 (June-July), alcohol from corn captured three percent of gasoline market within a year of its introduction in the market in the state of Iowa (33). Production of bio-gas from farm wastes is another opportunity in the farm sector for developing alternative energy sources.

Zeimetz (74), evaluating the potential of biomass grown on farms as an alternative source to fossil fuel and nuclear energy, points out that, "under the present technology, the cost of energy contained in biomass grown on energy farms is several times the current cost of energy contained in crude oil or coal." She estimates that, "currently, to produce one percent of the U.S. energy needs from biomass farming would require at least 10 million acres of good to very good quality land."

Thus, adjustments may be necessary in resource use and enterprise choice in the farm sector in response to rapidly changing energy situation. The degree and nature of adjustments will depend upon the severity of the energy crisis. In this study, an attempt is made to evaluate some of the impacts of: a) increase in energy price and 2) reduction in supply of energy to agricultural sector,

in the United States. As a subsidiary goal, the study also develops a quadratic programming model of U.S. crop sector for the purpose of evaluating the impacts of alternative energy situations. Specifically, following four different alternatives are analyzed in this study:

1. The low energy price scenario-characterized by nearly double of 1979 average prices for diesel, natural gas, electricity and LPG, and unrestricted supply of energy.
2. The high energy price scenario-characterized by nearly quadrupled 1979 average prices for diesel, natural gas, electricity and LPG, and unrestricted supply of energy.
3. The energy supply reduction on national basis - supply of energy to the farm sector is reduced by 10 percent of the energy used in the model under the low energy price scenario. The supply of energy is reduced on national basis. Energy prices are same as those under the low energy price scenario.
4. The energy supply reduction on regional basis - supply of energy to the farm sector is reduced by 10 percent of the energy used in the model under the low energy price scenario. The supply of energy is reduced on regional basis. Energy prices are the same as those under the low energy price scenario.

## CHAPTER II. THEORETICAL DEVELOPMENT OF THE MODEL AND METHODS

Agriculture in the United States is an industry that comes nearest to the economist's definition of perfectly competitive industry. Therefore, many economic studies assume a perfect competition model for agriculture.

Koopmans (34) defines competitive equilibrium as,  
 ". . . a balancing bundle of choices satisfying postulates 1-4 and a system of prices, one for each commodity, such that if all 'values' are computed at these prices,

- a. the choice of each consumer is preferred or equivalent to all other choices in his consumption set that are of equal or lesser value,
- b. the choice of each producer yields the maximum value attainable in his production set,
- c. the value of the commodities released by each resource holder is the maximum value attainable to him under postulate 4."

A bundle of choices for each commodity is defined by Koopmans (34) to be balanced, when "the net sum of all amounts chosen by producers and resource holders equals that of all amounts chosen by consumers." Koopmans' (34) four postulates referred to above, concern the behavior of decision-makers - the consumers, producers and resource holders.

"Postulate 1: There is a given number of decision makers, which can be sub-divided into 1 consumers, m

producers and  $p$  resource holders. There is a finite number  $n$  of commodities, sub-divided into types of labor and other commodities. Each decision maker makes one decision which consists in the choice of an amount of each commodity, that is, of a point in the commodity space.

Postulate 2: The point  $x^i$  chosen by the  $i^{\text{th}}$  consumer is constrained to a consumption set in which each point has a nonnegative coordinate for each commodity other than labor. The consumption set and the ordering on it are independent of the choices of other decision makers.

Postulate 3: The point  $y^j$  chosen by the  $j^{\text{th}}$  producer is constrained to the production set in which each point has a nonpositive coordinate for each type of labor. This set is independent of the choices of other decision makers.

Postulate 4: Each resource holder controls a non-negative quantity of each commodity which is not a type of labor, and chooses to release of each such commodity a nonnegative amount at most equal to what he holds."

The definitions of above 4 postulates are cited from Koopmans (34). Hall (26) notes that, partial competitive equilibrium is different from full competitive equilibrium in two respects. Firstly, not all prices are variable. Secondly, resources considered may have no alternative uses than postulated. In all the sector models of the economy, only partial competitive equilibrium is considered.

Development of market level (as opposed to firm level) models for a region as a whole, often is in the framework of competitive equilibrium. Mathematical models are formulated to simulate market behavior with the decentralization properties. That is, the market level equilibrium is consistent with the decision-making at the individual decision maker's level. Aggregate or market level models are based on "representative consumer" or "representative firm" concepts. The concept of competitive equilibrium or competitive market equilibrium can be stated in terms of prices and quantities of production and consumption as follows. In competitive market equilibrium,

a. Profits defined as difference between total revenue and payments (including rent) to all factors of production are zero for all producers.

b. Prices charged to consumers do not exceed the cost of production (including the rent charged to fixed factors of production).

c. Excess demand, defined as the difference between quantity demanded and quantity supplied at a given price level, is nonpositive in each market. If excess demand is negative then price of the commodity is zero.

d. Price differences across regions do not exceed the cost of transportation.

These conditions are of practical value in constructing

empirical models of competitive markets.

Quadratic Programming (QP) and QP Models of Competitive  
Spatial Equilibrium

Samuelson (48) formulated a programming problem in which the "net social pay off" was maximized and the resulting price and quantities of commodities traded satisfied the competitive spatial equilibrium conditions. Numerous applications of Samuelson's (48) work have appeared in the literature. In general, the applications of Samuelson's (48) formulation incorporated linear demand functions in the models rather than assuming that demand levels for commodities are exogenously specified. The sector models in linear programming framework assume fixed levels of demand.

Yaron (72) developed a model similar to Samuelson's (48) model, for numerous products with independent demands (that is, quantity demanded of a commodity is a function of its own price). Fox (23), and Schrader and King (49) incorporated linear demand functions and used an iterative algorithm to obtain equilibrium prices and quantities.

Takayama and Judge (59, 60, 61) in a series of studies formulated QP models to solve for competitive spatial equilibrium prices and quantities of commodities. In these studies by Takayama and Judge (59, 60, 61), linear demand and supply functions for commodities were used. In one of

their studies, Takayama and Judge (59) combined the activity analysis model of production and the linear demand functions.

Plessner (44), and Plessner and Heady (45) pointed out that, Takayama and Judge (60) formulation could not be extended to the case where the demand function coefficients are not symmetric for commodities in the model. Plessner (44), and Plessner and Heady (45) suggested an alternative formulation for the case where the demand function coefficients are asymmetric. Takayama and Judge (62) presented a reformulation of their model to include the cases of asymmetric demand coefficients.

Plessner's (44) work was further extended and applied in a series of studies by Hall (26), Stoecker (56), Chen (11), Meister et al. (40) and Olson et al. (43).

At this point, an elementary statement of the QP problem is presented and then various formulations of competitive equilibrium are reviewed.

A QP problem may be stated as follows (see Sposito (53)).

$$\text{Problem I: Maximize } z_1 = x'Dx - c'x \quad (2.1)$$

$$\text{such that } Ax \leq b \quad (2.2)$$

$$x \geq 0 \quad (2.3)$$



where

$x$  and  $c$  are  $(n \times 1)$  vectors;

$D$  is a symmetric negative semi-definite matrix;

$A$  is an  $(m \times n)$  matrix; and

$b$  is an  $(m \times 1)$  vector.

A dual for problem I (which may be called, "primal") may be stated as follows:

$$\text{Problem II: Minimize } z_2 = -x'Dx + b'\lambda \quad (2.4)$$

$$\text{such that } 2Dx - A'\lambda \leq c \quad (2.5)$$

$$x, \lambda \geq 0 \quad (2.6)$$

where

$x$ ,  $D$ ,  $b$ ,  $c$  and  $A$  are as defined in problem I, and  $\lambda$  is an  $(m \times 1)$  vector.

Note that, the requirement that  $D$  be symmetric is not a constraint from a purely QP point of view. If  $D$  is not symmetric, the objective functions can be rewritten as,

$$\text{Maximize } z_1 = x' \left( \frac{D+D'}{2} \right) x - c'x \quad \text{for problem I} \quad (2.7)$$

and

$$\text{Minimize } z_2 = -x' \left( \frac{D+D'}{2} \right) x + b'\lambda \quad \text{for problem II} \quad (2.8)$$

$\left( \frac{D+D'}{2} \right)$  is a symmetric matrix. However, the constraint set of the dual problem now contains  $\left( \frac{D+D'}{2} \right)$ , rather than  $D$ . At the optima, the vectors  $(x^*, \lambda^*)$  which solve the two problems, problems I and II, satisfy the following conditions:

$$(i) \quad x^{*'} D x^* - c' x^* = -x^{*'} D x^* + b' \lambda^* \quad (2.9)$$

$$(ii) \quad (A x^* - b) \leq 0 \quad (2.10)$$

$$(iii) \quad (A x^* - b)' \lambda^* = 0 \quad (2.11)$$

$$(iv) \quad (2 D x^* - A' \lambda^* - c) \leq 0 \quad (2.12)$$

$$(v) \quad (2 D x^* - A' \lambda^* - c)' x^* = 0 \quad (2.13)$$

$$(vi) \quad x^*, \lambda^* \geq 0 \quad (2.14)$$

The conditions (ii) to (v) are known as Kuhn-Tucker conditions for optimum (52). By combining the constraints of problems I and II Lagrangian constraint set is obtained. This is described by  $x, \lambda \geq 0$  and,

$$\begin{bmatrix} 0 & A \\ -A' & 2D \end{bmatrix} \begin{bmatrix} \lambda \\ x \end{bmatrix} \leq \begin{bmatrix} b \\ c \end{bmatrix}.$$

The constraint set of Problem I may be called, "initial constraint set". The QP algorithms make use of the Lagrangian constraint set in some form or the other.

With this description of QP problem, we now consider the representation of competitive market equilibrium by QP models. In this representation, we follow the convention of distinguishing the optimal levels of vectors by the superscript "\*".

Assumptions: 1. Let the inverse demand functions for  $n$  commodities be represented as

$$p = d_0 + \bar{D}q \quad (2.15)$$

where

$p$  is an  $(n \times 1)$  vector of nonnegative prices;

$q$  is an  $(n \times 1)$  vector of nonnegative quantities;

$d_0$  is an  $(n \times 1)$  vector; and

$\bar{D}$  is an  $(n \times n)$ , symmetric, and negative semi-definite matrix.

2. Let the production technology be represented by the input-output coefficient matrix  $B$ , and the activity levels and the limited levels of resources available be given by vectors  $x$  and  $b$ , respectively. The cost per unit of activity level is given by the elements of vector  $c$ .

Dimensions of the matrices and vectors described under assumption 2 are as follows:

$A$  is an  $(n \times n_1)$  matrix;

$B$  is an  $(m \times n_1)$  matrix;

$x$  and  $c$  are  $(n_1 \times 1)$  vectors; and

$b$  is an  $(m \times 1)$  vector.

$$\text{Problem III: Maximize } z_3 = d'_0 q + \frac{1}{2} q' \bar{D} q - c'x \quad (2.16)$$

such that,

$$Bx \leq b \quad (2.17)$$

$$q - Ax \leq 0 \quad (2.18)$$

$$q, x \geq 0 \quad (2.19)$$

The Lagrangian constraint set for problem III is,

$$\begin{bmatrix} 0 & 0 & 0 & B \\ 0 & 0 & I & -A \\ 0 & -I & D & 0 \\ -B' & A & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ q \\ x \end{bmatrix} \leq \begin{bmatrix} b \\ 0 \\ -d_0 \\ c \end{bmatrix}$$

Where  $\lambda_1$  and  $\lambda_2$  are  $(n \times 1)$  vectors and  $I$  is an identity matrix of dimension  $(n \times n)$ .

Using Kuhn-Tucker conditions, the Lagrangian multipliers  $\lambda_1$  and  $\lambda_2$  can be given economic interpretations. Consider the constraint (2.17)

$$Bx \leq b$$

reflecting the condition of limited resource availability.

The Kuhn-Tucker condition associated with constraint (2.17) is

$$(Bx^* - b)' \lambda_1^* = 0 \quad (2.20)$$

Combining (2.17) and (2.20), it is clear that  $\lambda_1$  has

the standard interpretation of "shadow price" of resources. If a resource is not entirely exhausted, the resource has zero price; otherwise it has a nonnegative price. Consider the constraint (2.18)

$$q - Ax \leq 0$$

and the associated Kuhn-Tucker condition

$$(q^* - Ax^*)' \lambda_2^* = 0 \quad (2.21)$$

which relate to supply and pricing in the output sector. Constraint (2.18) implies that supply of output should at least be as great as demand. Constraint (2.21) implies that only when supply exactly equals demand,  $\lambda_2$  may be positive. Otherwise,  $\lambda_2$  is zero. Thus,  $\lambda_2$  may be interpreted as the vector of prices of commodities in vector  $q$ .

The constraints in the Lagrangian set and related to the dual of problem III are,

$$\lambda_2 - (d_0 + \bar{D}q) \leq 0 \quad (2.22)$$

and

$$[\lambda_2^* - (d_0 + \bar{D}q^*)]' q^* = 0 \quad (2.23)$$

The constraint (2.22) defines  $\lambda_2$  to be at most equal to the prices of commodities corresponding to output levels in the vector  $q$ . The constraint (2.23) implies that output

level of a commodity is positive only when the corresponding element in  $\lambda_2$  exactly equals the price of the commodity given by the demand functions. Finally, the constraints

$$-B'\lambda_1 + A'\lambda_2 \leq c \quad (2.24)$$

and

$$(-B'\lambda_1^* + A'\lambda_2^* - c)'x^* = 0 \quad (2.25)$$

insure that, an activity level will be positive only for zero net profit. Thus, solution to problem III, when considered along with the dual problem as well, is consistent with partial competitive market equilibrium conditions.

In economic literature, the value of objective function in problem III is recognized as the sum of consumer and producer surpluses (17).

In specifying problem III, a crucial assumption of "symmetric demand matrix" has been made. There are two primary motivations for delving further into the above assumption: 1. to determine the basis and scope of enquiry using a particular type of demand functions; and 2. to determine the merits and demerits of using a particular type of demand functions. We now discuss these two points.

Regarding the first motivation listed above, there is no restriction in principle for the consumer demand matrix to be symmetric. When the demand matrix  $\bar{D}$  in the set of linear

demand functions,

$$q = d_0 + \bar{D}p \quad (2.26)$$

where

$q$  and  $p$  are  $(n \times 1)$  vectors of quantities and prices;

$d_0$  is an  $(n \times 1)$  vector; and

$\bar{D}$  is an  $(n \times n)$  matrix,

is asymmetric, Stoecker (56) notes that demand functions may exist as a set of nonexact differential equations.

However, when a set of consumer demand functions are estimated, almost as a rule, certain restrictions are placed on parameters. These restrictions are derived from the consumer theory which postulates a consumer behavior of maximization of a utility function subject to a budget constraint. For examples of application of such restrictions see Brandow (9) and George and King (24).

The assumption of an ordinal, neoclassical, utility function for the consumer is used in many applied and theoretical economic studies. Allen (3) and Silberberg (50) point out that when the demand matrix in a set of more than two linear demand functions is asymmetric, there exists no utility function which gives rise to the above demand functions under the postulate of utility maximization subject to budget constraint.

Hence, for interpretation beyond that of mere empirical

relationships between prices and quantities, it is necessary to assume or impose the symmetry of demand matrix. While the very interpretation of the sum of areas between consumers' demand and producers' supply curves as a measure of societal welfare is controversial (see Harberger (28)), attempts have been made by economists to devise measures of welfare even when no unique social welfare function (derived from individual consumer utility functions) exists. A survey of these issues can be found in Mann (38).

Regarding the use of linear demand functions it must be noted that linear relationships can be more easily used than nonlinear relationships in applied work. Linear demand functions may also represent adequately the observed relationship between prices and quantities, within a narrow range of data. When more accurate representation is desired, other functional forms are appropriate. From the viewpoint of representing demand structures in a mathematical programming model, linear demand functions are most convenient to use. Once the set of linear demand functions are chosen, a formulation appropriate for obtaining competitive equilibrium solutions must also be chosen.

Noting that demand matrix  $D$  may be asymmetric in many instances, where no attempt is made to associate behavioral assumptions of utility maximization, Plessner and Heady (45) proposed an alternative formulation of the



problem. For this case, let the set of demand functions be as in Equation (2.26).

Problem IV: (P-H form)

$$\text{Maximize } z_4 = d'_0 q + p' \bar{D} p - b'u - c'x \quad (2.27)$$

such that

$$d_0 + \bar{D} p - Ax \leq 0 \quad (2.28)$$

$$Bx \leq b \quad (2.29)$$

$$A'p - B'u \leq c \quad (2.30)$$

$$p, u, x \geq 0 \quad (2.31)$$

Using the results of studies by Dorn (16) and Hanson (27), Plessner and Heady (45) showed that problem IV is self-dual. As compared to problem III, problem IV has an additional set of  $n_1$  constraints. The Lagrangian constraint set of the P-H form has a large number of additional constraints as compared to the Lagrangian constraint set for problem III. Stoecker (56) and Takayama and Judge (62) have independently shown that, in solving the P-H form of the competitive equilibrium problem, one needs to consider only the initial constraint set of the P-H form. Stoecker (56) characterized the P-H form as a self-dual, initial constraint set corner point solution problem and has shown that, P-H form also corresponds to the class of negative semidefinite programming problems dealt with by Cottle and Dantzig (13). But note that the initial

constraint set of the P-H form is different from that of problem III. In fact, the Lagrangian constraint set of problem III corresponds more closely to the initial constraint set of P-H form, in terms of the number of constraints involved.

The fact that initial constraint sets are different in P-H form and problem III is important, because if a QP computer algorithm is used to solve problem III, then the choice between P-H form and problem III is not crucial from the standpoint of computational efficiency. From Stoecker's (56) results, we have that the initial constraint set of P-H problem is the only set of linear constraints in the model. But if separable programming is used to solve problems of the type of problem III, then the choice between problem III formulation and P-H form is crucial. When demand matrix  $\bar{D}$  is asymmetric, use of problem III along with separable programming is inappropriate. In such a case, choice of P-H form is the appropriate one.

The necessity of having to use P-H form when the demand matrix is asymmetric, considerably limits the scope of problems that can be handled with economy in computations. Carey (10) has suggested an iterative approach in the cases of "factor integrable" demand functions. He also suggests iterative procedures for the general case. Plessner and Heady (45) also suggest an iterative method

of solution. But iterative methods also considerably limit the scope of problems that can be handled with economy in computations. Hence, use of symmetric matrices would have certain advantages. But in the final analysis, how "well" symmetric matrices perform in representing the empirical data should be the determining factor in the choice of a particular set of demand functions. In the present study, a set of linear demand functions giving rise to symmetric demand matrix was chosen.

#### Separable Programming and Approximate Solutions to Nonlinear Models

Separable programming has been used in a number of studies in applied economics. The study of an ore purchasing problem, by Beal et al. (6) is a good example in this area. Theory of separable programming is developed in Hadley (25). Martin (39) discussed various formulations of separable programming where commodity demand functions and resource supply functions are incorporated in programming models. Yaron and Heady (73) discussed the same idea in the context of a broader problem in which they consider nonlinear models with separable objective functions. Duloy and Norton (17) applied separable programming to the interregional competition model of Mexican agriculture. Taylor and Frohberg (64) used separable programming in

evaluating impacts of banning pesticides and fertilizers, and erosion control in the corn belt region of the United States in a competition model incorporating linear demand functions for corn and soybeans. Swanson and Taylor (58) used a similar model as Taylor and Frohberg (64), to evaluate the impacts of increases in energy prices on agriculture in the corn belt region. Taylor et al. (63) developed a separable programming model of interregional competition model of U.S. agriculture in which linear demand functions for commodities were incorporated. More recently, Boggess (8) has used separable programming to solve a quadratic programming model of U.S. agriculture.

Nonlinearity in programming models may arise either in the objective function to be optimized or in the constraints of the models. Algorithms to obtain exact solutions are available only for certain well-defined classes of problems. For example, computer algorithms exist to solve even large sized QP models, which are a class of nonlinear models. Separable programming is one technique which may be used to solve a class of nonlinear programming problems.

The class of nonlinear programming problems which can be solved with separable programming consists of nonlinear models in which only separable functions are involved. The general form of the separable programming problem is (see

Hadley (25)),

Problem V:

$$\text{Maximize } z_5 = \sum_{j=1}^n f_j(x_j) \quad (2.32)$$

$$\text{such that } \sum_{j=1}^n g_{ij}(x_j) \{ \leq = \geq \} b_i \forall i=1,2,\dots,m \quad (2.33)$$

The functions  $f_j$  and  $g_{ij}$  have only one variable  $x_j$  as argument. Hence they are separable functions. There are mainly two methods of solving problem V via separable programming. Hadley (25) defines them as: a) the  $\lambda$ -approximating form and b) the  $\delta$ -approximating form. In this section only  $\delta$ -approximating form (or the delta method) will be discussed. Duloy and Norton (17) used a variant of  $\lambda$ -approximating form in applying separable programming to interregional competition model of Mexican agriculture. In illustrating the delta method only the objective function of the problem is assumed to be nonlinear. Constraints are assumed to be linear.

Example:

$$\text{Problem VI: Maximize } z_6 = \sum_{j=1}^n f_j(x_j) \quad (2.34)$$

$$\text{such that } \sum_{j=1}^n a_{ij}x_j \leq b_i \forall i=1,2,\dots,m \quad (2.35)$$

$$x_j \geq 0 \forall j=1,2,\dots,n \quad (2.36)$$

Also,  $f_j$  ( $\forall j=1,2,\dots,n$ ) is a concave function of  $x_j$ .

Now suppose the range of  $x_j$  is divided into  $r_j$  segments and for each corresponding  $x_{kj}$  ( $k^{\text{th}}$  segment of  $x_j$ ), the value of  $f_j$  is given by  $f_{kj}$  for all  $k=1,2,\dots,r_j$ .

Let,

$$\Delta f_{kj} = f_{kj} - f_{k-1,j}; \text{ and} \quad (2.37)$$

$$\Delta x_{kj} = x_{kj} - x_{k-1,j} \quad \forall k=1,2,\dots,r_j, \\ j=1,2,\dots,n. \quad (2.38)$$

Then, if  $x_j$  lies in the interval  $x_{k-1,j} \leq x_j \leq x_{kj}$  we can write,

$$x_j = x_{k-1,j} + \delta_{kj} (\Delta x_{kj}) \quad (2.39)$$

where

$$\delta_{kj} = (\Delta x_{kj})^{-1} \cdot (x_j - x_{k-1,j}) \quad (2.40)$$

Furthermore, for  $x_j$  in the interval given before,

$$0 \leq \delta_{kj} \leq 1. \quad (2.41)$$

Then the approximate value of  $f_j$  corresponding to  $x_j$  can be written as

$$\hat{f}_j = f_{k-1,j} + \delta_{kj} \cdot (\Delta f_{kj}) \quad (2.42)$$

Hadley (25) notes that, this is a linear or polygonal approximation of the true function. Consider the restriction that if  $\delta_{kj} > 0$  then  $\delta_{uj} = 1$  for  $u = 1,2,\dots,k-1$ .

We can write,

$$f_{k-1,j} = \sum_{u=1}^{k-1} (\Delta f_{uj}) \cdot \delta_{uj} + f_{0j} \quad (2.43)$$

where

$f_{oj}$  is the value of  $f_j$  when  $x_j = 0$ ,  
(or  $f_{oj}$  may correspond to some selected value  
of  $x_j > 0$ ).

Note that, if  $0 < \delta_{kj} < 1$ , then  $\delta_{uj} = 0$  for  $u > k$ .

The maximization problem (problem VI) can be rewritten  
as follows:

Problem VIa:

$$\text{Maximize } z_7 = \sum_{j=1}^n \left( \sum_{k=1}^{r_j} (\Delta f_{kj}) \cdot \delta_{kj} + f_{oj} \right) \quad (2.44)$$

$$\text{such that } \sum_{j=1}^n \sum_{k=1}^{r_j} a_{ij} (\Delta x_{kj}) \cdot \delta_{kj} \leq b_i \quad \forall i=1,2,\dots,m \quad (2.45)$$

$$0 \leq \delta_{kj} \leq 1; \text{ and} \quad (2.46)$$

if

$$\begin{aligned} 0 < \delta_{kj} < 1, \text{ then } \delta_{uj} &= 1 \text{ for} \\ u &= 1, 2, \dots, k-1, \text{ and,} \\ \delta_{vj} &= 0 \text{ for } v > k. \end{aligned} \quad (2.47)$$

The problem VIa differs from a typical linear programming problem only with respect to the "restricted entry" of  $\delta_{kj}$  in the basis.

#### Transformation of variables to obtain separability

One of the prerequisites for applying separable programming to nonlinear models is that the functions be separable. It is possible however, to convert certain

nonseparable functions into separable ones by defining some new variables. Hadley (25) provides a discussion on this aspect. Here, transformation of variables to handle certain type of nonseparability is considered. Let a set of linear demand functions for  $n$  commodities be

$$q = \begin{matrix} & d_0 & + & \bar{D}p \\ (nx1) & (nx1) & & (nxn)(nx1) \end{matrix} \quad (2.48)$$

In QP formulation of competitive equilibrium one often has the total revenue, or sum of area under the demand curves as part of the objective function. Consider the total revenue expression,

$$p'q = p'd_0 + p'\bar{D}p \quad (2.49)$$

or,

$$p'q = \sum_{i=1}^n p_i d_i + \sum_i \sum_j a_{ij} p_i p_j$$

where  $a_{ij}$  are elements of the matrix  $\bar{D}$ , and  $d_i$  are elements of vector  $d_0$ . Thus, the objective function contains variables of second degree some of which are separable ( $a_{ii}p_i^2$ ) and some are not ( $a_{ij}p_i p_j$ ).

#### Hadley's transformations:

The nonseparable product terms such as  $p_i p_j$  ( $i \neq j$ ) can be redefined in terms two new variables to yield separable terms. Hadley (25) shows that,



$$p_i p_j = s_{lij}^2 - s_{2ij}^2 \quad (2.50)$$

where

$$s_{lij} = \left(\frac{p_i + p_j}{2}\right) \quad \text{and} \quad s_{2ij} = \left(\frac{p_i - p_j}{2}\right)$$

Thus we can write,

$$\begin{aligned} p'q = & \sum_i p_i d_i + \sum_i a_{ii} p_i^2 + \sum_i \sum_{j \neq i} (a_{ij} + a_{ji}) (s_{lij}^2) \\ & - \sum_i \sum_{j \neq i} (a_{ij} + a_{ji}) s_{2ij}^2 \end{aligned} \quad (2.51)$$

With the restriction that  $s_{lij}$  and  $s_{2ij}$  are appropriately defined. However, there is an important limitation to this approach. For  $n$  commodities, there will be  $\frac{n(n-1)}{2}$  cross-product terms in the objective function. For each cross-product term, if 2 new variables are defined  $n(n-1)$  additional restrictions will be required for the programming problem.

Eigenvalue transformations      Any real symmetric matrix  $\bar{D}$  can be expressed as follows (see Intrilligator (32)):

$$\bar{D} = TVT' \quad (2.42)$$

where

$V$  = a diagonal matrix ( $n \times n$  size) with eigenvalues of  $\bar{D}$  on the diagonal,

$T$  = matrix ( $n \times n$  size) of eigenvectors corresponding to the eigenvalues in matrix  $V$ .

Thus, the total revenue may be written as,

$$p'q = d_0'p + p'\bar{D}p = d_0'p + p'TVT'p \quad (2.53)$$

Note that the demand matrix  $D$  is converted into a symmetric matrix without affecting the value of  $p'q$ . Furthermore, the matrix  $D$  contains only real elements and the derivation above in terms of eigenvalues and eigenvectors can be applied.

$$\text{Let } T'p = t \quad (2.54)$$

then

$$p'q = d_0'p + p'TVT'p = d_0'p + t'Vt \quad (2.55)$$

$$= \sum_{i=1}^n d_i p_i + \sum_{i=1}^n v_{ii} t_i^2 \quad (2.56)$$

where

$v_{ii}$  is the diagonal element in  $i^{\text{th}}$  row of the matrix  $V$ ,

$t_i$  is the linear combination of  $p_i$ 's, defined by  $t = T'p$ .

Thus, the  $\frac{n(n-1)}{2}$  cross-product terms can be defined by only  $n$  new variables. Accordingly, instead of  $n(n-1)$  additional restrictions, only  $n$  restrictions are needed. While in principle, the eigenvalue approach requires definition of only  $n$  new variables, more than  $n$  constraints may be needed in the programming matrix. This is due to the possibility that the special variables which are a linear combination of prices may not involve all coefficients of the same sign. There are other methods which may be used to

transform the quadratic form. For example, Ayres (5) describes the Legendre's transformation of quadratic forms. Because of the relative ease with which coefficients could be generated, eigenvalue transformations were used in this study. The eigenvalues and eigenvectors used in this study are reported in the Appendix.

### CHAPTER III. THE MODEL AND ALTERNATIVES ANALYZED

#### The Model

The model used in the present study is a modified version of the linear programming models used at the Center for Agricultural and Rural Development (CARD). For a documentation of these models see Meister and Nicol (41) and Huang et al. (31). The present model is also based in several respects on the studies by Dvoskin and Heady (18), Dvoskin et al. (19), and Boggess (8). There are several assumptions fundamental in modeling agricultural production with an activity analysis type of model. Agrawal and Heady (1) list these assumptions (in a general context) as follows:

1. Additivity of resources and activities;
2. linearity of the objective function;
3. nonnegativity of the decision variables;
4. divisibility of activities and resources;
5. finiteness of the activities and resource restrictions;
6. proportionality of activity levels to resources;  
and
7. single-value expectations.

Except for assumption (2) above, that of linearity of the objective function, all other assumptions are implicit

in the present model. The objective function in the present model is a quadratic expression. Many additional assumptions need to be made regarding the nature of markets involved, mobility of the resources and other behavioral assumptions in constructing a particular programming model of agricultural sector. These assumptions are not discussed here; Takayama and Judge (59) present a set of assumptions which are relevant for the present model also.

The mainland United States is divided into 105 agronomically homogeneous producing areas (PA's). The boundaries of these PA's are shown in Figure 3.1. The PA's are aggregated into nine consuming or market regions (MR's). The nine MR's are also aggregations of 28 MR's used in the study by Dvoskin and Heady (18). The two sets of MR's for comparison and reference, are shown in Figures 3.2 and 3.3.

Crop production and land base are defined at the PA level. Crop demand is defined at MR level. The central cities shown in each MR in Figure 3.2 act as demand centers with transportation activities for crops defined among them. The model has four major components: 1) crop demand; 2) crop supply; 3) input demand; and 4) input supply. Livestock sector is an exogenous component of the model.

The model simulates production of feed grains (barley, corn, oats and sorghum), wheat, soybean, cotton, hay

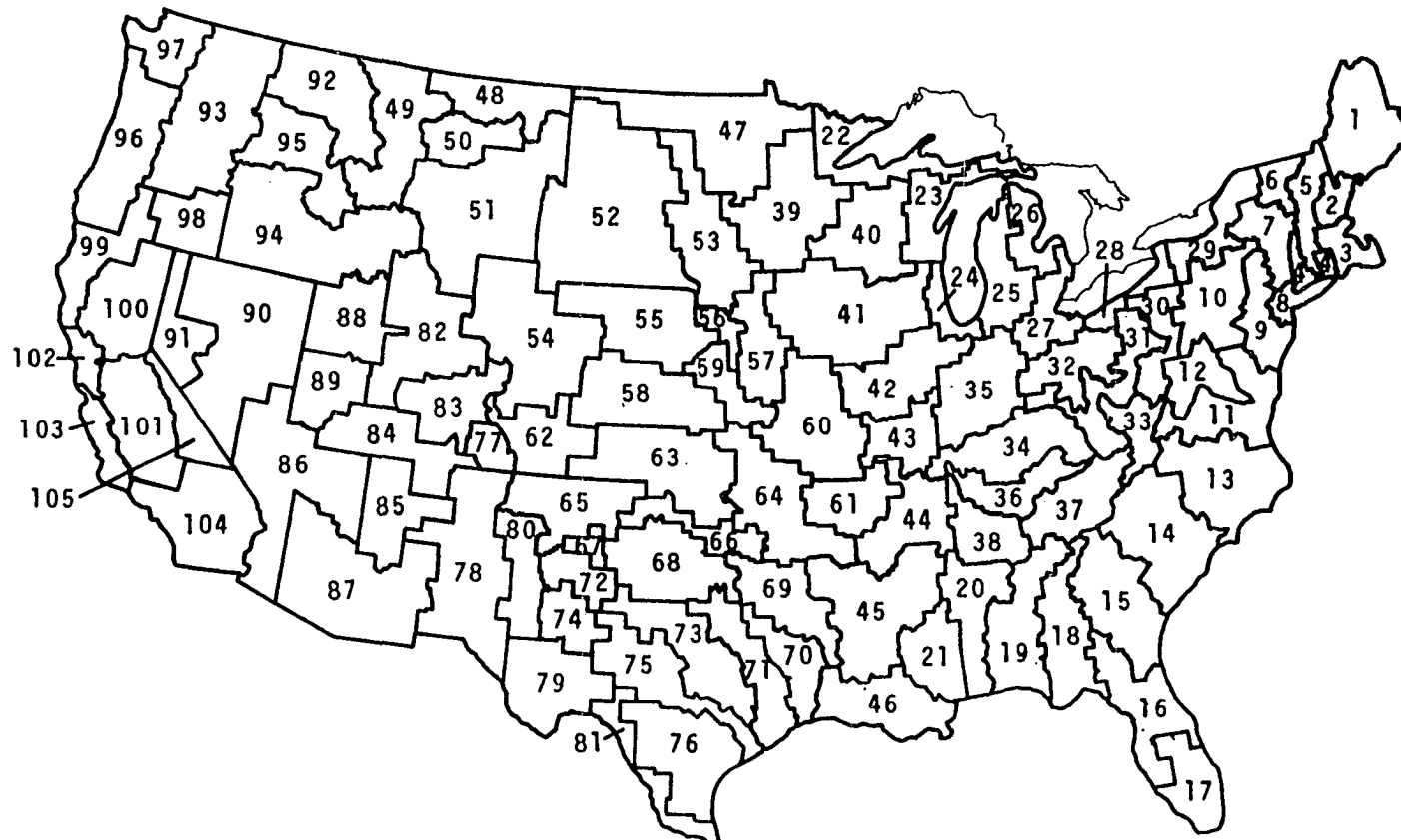


Figure 3.1. The 105 producing areas in the model

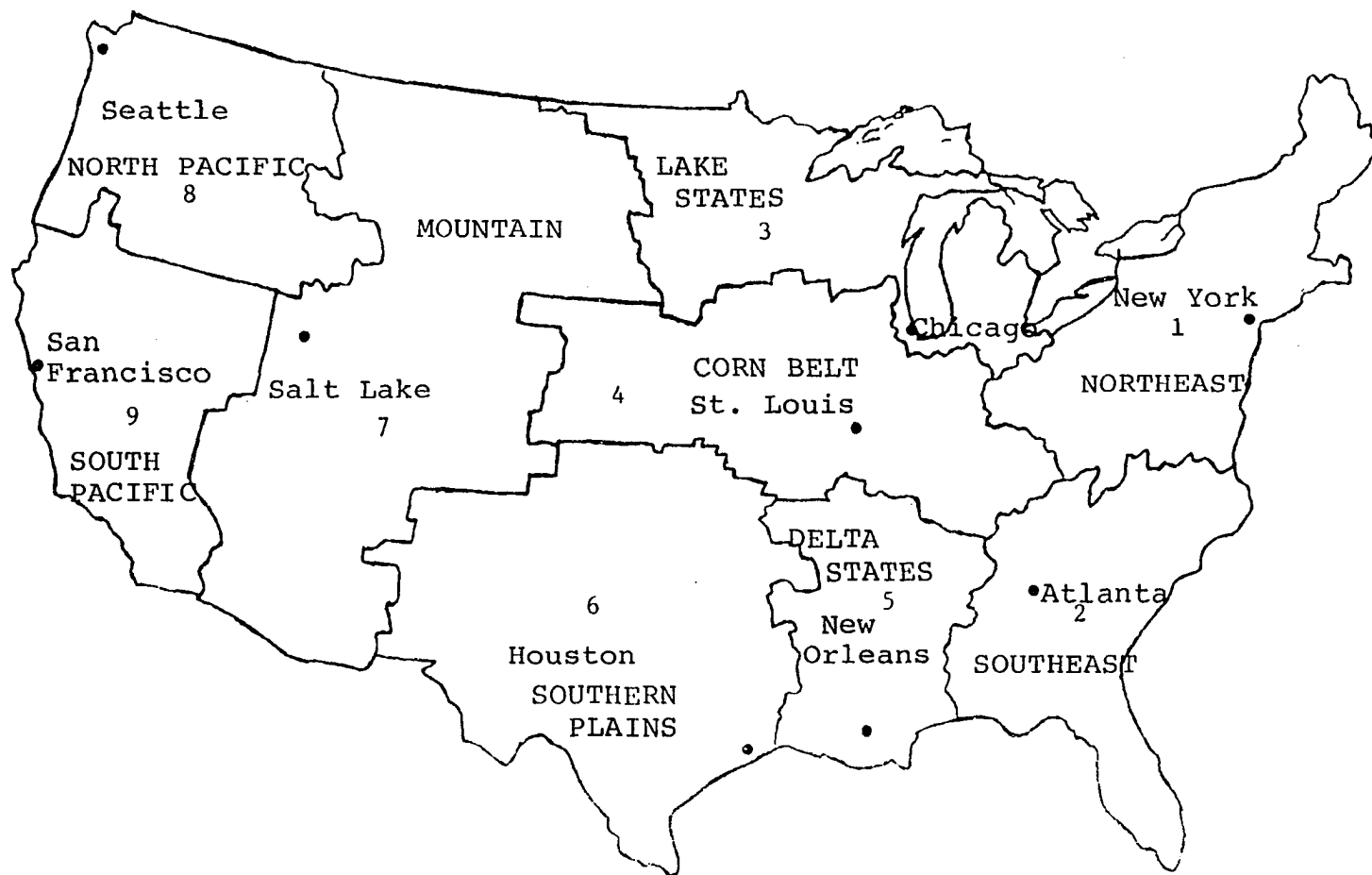


Figure 3.2. The nine market regions with the central cities indicated

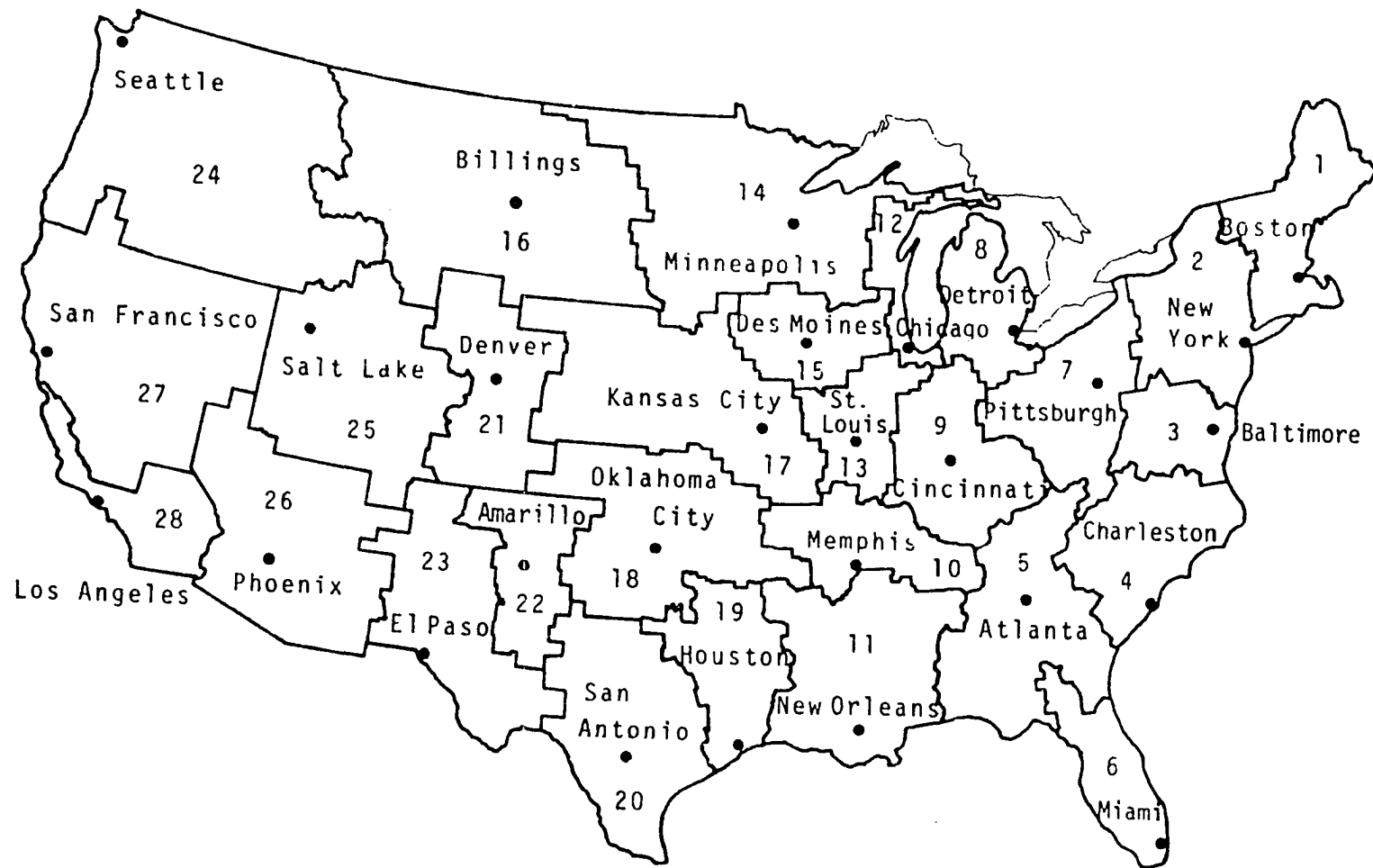


Figure 3.3. The 28 market regions with central cities indicated



(legume and nonlegume) and silage (corn and sorghum). The inputs explicitly considered in the model are, land (dry and irrigated), water (surface and ground), nitrogen (commercial fertilizer and livestock) and energy (diesel, electricity, natural gas and liquid petroleum gas). The present study has a time reference of the year 1985. Hence, all the estimations are projections to the year 1985.

### The Crop Demand Sector

Demand for crops is defined at MR level for all except cotton. For cotton, the lint demand is specified at national level. Based on the type of demand specified there are two major classes of crops: 1) crop demand entirely predetermined to the model; and 2) crop demand which is partly predetermined. For a given commodity, demand originates from several sectors. The major consideration in specifying a particular demand structure for a commodity was availability of data.

#### Feed grains and wheat

Barley, corn, oats and sorghum are commonly referred to as feed grains in this study. But these crops are also used for purposes other than livestock feeding. Feed grains are used for domestic food and industrial uses and are also exported. Similarly, wheat is used for food and industrial

uses, fed to livestock and also exported. For barley, corn, oats and wheat demand for domestic food and industrial uses is given by a set of linear demand functions developed from the elasticity estimates of Brandow (9). For sorghum, demand for food and industrial uses is specified exogenously. Demand function for a commodity for food and industrial uses is of the following form:

$$q_j = d_j + \sum_j b_{ij} p_j \quad (3.1)$$

where

$q_j$  = quantity demanded of  $j^{\text{th}}$  commodity;

$p_j$  = price of  $j^{\text{th}}$  commodity;

$d_j$  = intercept of demand equation of  $j^{\text{th}}$  commodity; and

$b_{ij}$  = slope coefficients of demand equation for  $j^{\text{th}}$  crop.

Brandow (9) has estimated farm level price elasticities of demand for a large number of agricultural products. The price elasticities of demand are of the form

$$e_{ij} = \frac{\delta \ln q_i}{\delta \ln p_j} = \frac{\delta q_i}{\delta p_j} \cdot \frac{p_j}{q_i} \quad (3.2)$$

where

$$\frac{\delta q_i}{\delta p_j}$$

is the partial derivative of  $q_i$  with respect to  $p_j$ ;

$e_{ij}$  = farm level price elasticity of demand for  $i^{\text{th}}$  commodity with respect to price of  $j^{\text{th}}$  commodity;

$p_j$  = farm level price of  $j^{\text{th}}$  commodity for a given time period; and

$q_i$  = quantity demanded of  $i^{\text{th}}$  commodity for a given time period.

Thus in Equation (3.1), the coefficients  $b_{ij}$  can be written as,

$$b_{ij} = \frac{\delta q_i}{\delta p_j} \quad (3.3)$$

or,

$$b_{ij} = e_{ij} \frac{q_i}{p_j} \quad (3.4)$$

The quantities  $q_i$  and prices  $p_j$  need to be chosen in order to obtain the  $b_{ij}$  coefficients from price elasticities of demand.

The elasticity coefficients shown in Table 3.1 are converted into  $b_{ij}$  coefficients using quantities and prices shown in Table 3.2. The symmetric slope coefficients are obtained as  $(b_{ij} + b_{ji}) \times 0.5$ . The national level coefficients are converted into per capita basis by dividing the national level  $b_{ij}$  by the estimated 1963-65 average U.S. (mainland) population of 169.1 million. Finally, coefficients for 1985 are generated by multiplying the per capita  $b_{ij}$  coefficients with an estimated population base of 233.2 million (71). The national level  $b_{ij}$  coefficients for year 1985 are shown in Table 3.3.

The intercepts for demand equations are obtained in a manner described by Stoecker (56). The steps are,

Table 3.1. Price elasticity coefficients of demand for domestic food and industrial uses (Brandow (9))

1 percent change in price of	Percentage change in quantity of:				
	Barley	Corn	Oats	Wheat	Soybean oil
Barley	-0.073700	0.000100	0.000006	0.000200	0.000845
Corn	0.000022	-0.033200	0.000100	0.003800	0.000006
Oats	0.000006	0.000300	-0.007000	0.001100	0.000105
Wheat	0.000024	0.001200	0.000100	-0.021400	0.000377
Soybean oil	0.000041	0.000178	0.000015	0.000595	-4.389800

Table 3.2. Quantities and prices used to obtain slope coefficients of demand functions (USDA (66, 67))

	Quantities <sup>a</sup> (millions)				Prices <sup>b</sup> (dollars)			
	1963	1964	1965	Average	1963	1964	1965	Average
Barley	98	102	105	102	1.59	1.65	1.74	1.66
Corn	340	349	360	350	1.97	2.03	1.98	1.99
Oats	45	45	45	45	1.10	1.10	1.06	1.09
Wheat	503	501	502	502	3.27	2.38	2.30	2.65
Soybean oil	4058	4069	4687	4271	0.41	0.42	0.40	0.41

<sup>a</sup>Units for barley, corn, oats and wheat are bushels and for soybean oil units are pounds.

<sup>b</sup>Units for barley, corn, oats and wheat are dollars per bu and for soybean oil units are dollars per pound.

Table 3.3. The slope coefficients of demand functions

A unit change in price of	Change in quantity of:				
	Barley	Corn	Oats	Wheat	Soybean oil
	----- (million bu) -----				(million cwt)
Barley	-5.58444	0.06063	0.00045	.00921	0.00135
Corn	0.06063	-7.20075	0.02402	0.049613	0.00239
Oats	0.00045	0.02402	-0.035633	0.03999	0.00043
Wheat	0.00921	0.49613	0.03999	-4.99911	0.00874
Soybean oil	0.00135	0.00239	0.00043	0.00874	-5.59001

$$1. \text{ Obtain } E_{ij} = q_{ij} - \sum_{k=1}^n b_{ik} p_{kj}. \quad (3.5)$$

Using annual data on quantities ( $q_{ij}$ ) used of the  $i^{\text{th}}$  commodity for domestic food and industrial uses on a per capita basis in  $j^{\text{th}}$  year. The  $b_{ik}$ 's are coefficients of Table 3.2. Thus, the  $E_{ij}$ 's are residuals. Time-series data on quantities and prices from 1954 to 1976 are used.

2. Fit following regressions:

$$E_{ij} = a_{0i} + a_{1i}T_j + u_{ij} \quad (3.6)$$

and

$$E_{ij} = a_{0i} + a_{1i}T_j + a_{2i}T_j^2 + u_{ij} \quad (3.7)$$

The variable  $T$  is a trend variable with  $T_{1954} = 1$ . When autocorrelation of residuals  $u_{ij}$  is a severe problem as indicated by the Durbin-Watson statistic, the estimates of  $a_{ij}$  are corrected for the resulting bias. The criterion used for selecting a particular functional form is the mean square error (MSE) for the regression model and the model's ability to predict reasonable levels of intercept.

3. Predict the per capita level estimates of the intercepts,  $d_j$ 's, by setting  $T_{1985} = 32$  in the equations selected in step 2, and convert the intercept estimates to national level using the population level of 233.2 million. The

selected intercept equations and intercepts estimated for 1985 are reported in Table 3.4.

From the national level, the demand equations were disaggregated into MR level using the population proportion projections of OBERS (71) for the year 1985. The population proportions used are reported in Table 3.5. The set of demand equations in matrix form at MR level is of the following form:

$$q^k = d_0^k + \bar{D}^k p^k \quad (3.8)$$

where superscript  $k$  denotes the  $k^{\text{th}}$  MR ( $k = 1, 2, \dots, 9$ ) and  $q$ ,  $d_0$ ,  $\bar{D}$  and  $p$  are as previously described. The inverse demand functions for use in the model are obtained as follows

$$p^k = -(D^k)^{-1} \cdot d_0^k + (\bar{D}^k)^{-1} \cdot q^k \quad (3.9)$$

This completes the description of the demand for barley, corn, oats and wheat for domestic food and industrial uses. Livestock feed demand for feed grains and export levels for 1985 are obtained from Quance et al. (46). Livestock feed demand is represented in corn equivalents at MR level. Barley, corn, oats, sorghum or wheat may satisfy part or all of feed grain demand at each MR. The feed grain demand and export demand used in this study are reported in Table 3.6.



Table 3.4. Estimates of parameters of selected intercept equations<sup>a</sup>

Commodity	$a_0$	$a_1$	$a_2$	MSE	R square	Number of observations	D-W <sup>b</sup>	Predicted intercept (for 1985, T=32) <sup>c</sup>	
								Per capita	Total
Barley	0.7187*** (0.03)	0.0009 (0.002)	-	0.0003	0.118	21	-	0.7483	174.4966
Corn	1.5632***	0.0276***	-	0.0031	0.7498	21	-	2.4471	570.6628
Oats	0.9966*** (0.02)	-0.0512*** (0.004)	0.0012*** (0.0002)	0.0004	0.9708	21	-	0.5329	124.2676
Wheat	3.5874*** (0.03)	-0.0684*** (0.01)	0.0019*** (0.0003)	0.0021	0.9449	21	1.55	3.3108	772.0786
Soybean oil	107.8303*** (5.79)	-0.8058 (1.40)	0.0659 (0.07)	53.1396	0.1506	18	0.59	134.1083	31274.0550

<sup>a</sup>Numbers in parentheses are estimated standard errors of estimates of regression coefficients directly above them.

<sup>b</sup>D-W statistic is reported only for ordinary least squares equations. For others, equations are corrected for autocorrelation.

<sup>c</sup>Intercept for soybean oil is predicted with T=27. At T=32, soybean oil intercept is unacceptably high. Observations from 1973-1975 were delted in estimating soybean oil intercept as these observations were considered as outliers for the data set used.

\*\*\* Indicates the regression coefficient is statistically significant at  $\alpha=0.01$ .

Table 3.5. Projected population proportions by market region, for 1985 (United States Water Resources Council (71))

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Market region	Proportion of population in 1985
<hr/>	
Northeast	0.336405
Southeast	0.116891
Lake states	0.134532
Corn belt	0.105629
Delta states	0.051003
Southern plains	0.086289
Mountain region	0.035696
North Pacific	0.028865
South Pacific	0.104690
 TOTAL	 1.0

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Table 3.6. Livestock demand for feed grains and export demand for grains and soybean oil  
(Quance et al. (46))

Market region	Livestock demand for feed grains <sup>a</sup> (mill bu)	Export Demand					
		Barley (mill bu)	Corn (mill bu)	Oats (mill bu)	Wheat (mill bu)	Sorghum <sup>b</sup> (mill bu)	Soybean <sup>c</sup> (mill cwt)
Northeast	562.73	0.02	148.49	0.49	32.30	1.98	40.98
Southeast	534.35	0.99	277.05	0.62	106.50	0.74	155.72
Lake states	792.09	20.03	124.22	9.28	119.48	0.79	31.96
Corn belt	1821.06	0	0	0	0	1.04	62.18
Delta states	320.91	1.57	965.16	0	164.61	11.01	136.02
Southern plains	729.05	0	87.58	1.46	600.92	337.90	13.28
Mountain states	334.02	0.24	0	0	0	0.25	31.98
North Pacific	115.26	16.36	0.48	0.28	411.67	0.25	3.04
South Pacific	313.40	0	0	0	0	10.99	10.98
TOTAL <sup>d</sup>	5552.87	39.21	1602.98	12.13	1435.48	365.00	486.14

<sup>a</sup>Expressed in corn equivalent.

<sup>b</sup>Includes demand for food and industrial uses.

<sup>c</sup>Includes export of soybean oil; expressed in soybean oil meal equivalent.

<sup>d</sup>Total may not exactly equal to the sum of regional numbers due to rounding error.

### Soybeans

Soybeans are used to obtain soybean oil, soybean oil meal and for export. Soybean oil is used as a vegetable oil in food preparations and part of oil produced is also exported. The domestic demand for soybean oil is represented by a linear demand function of the type described in the case of barley, corn, oats and wheat. The coefficients of demand function for soybean oil are given in Tables 3.4 and 3.5.

Exports of soybean oil are obtained in a simple manner with an estimated regression equation relating U.S. exports of soybean oil to the time variable (for the period 1960-1973). The selected estimated equation is,

$$\ln \text{ exports (soybean oil) } = 6.7851^{***} + 0.1555^{***} \ln T \quad (3.10)$$

(0.14)                      (0.07)

$$\text{MSE} = 0.0429 \quad F = 5.661 \quad R^2 = 0.3398 \quad n = 13$$

where  $\ln$  refers to natural logarithm.

The predicted national level exports of soybean oil is disaggregated into MR level using historical exports of soybeans in each MR. Finally, demand for soybeans as beans is obtained from Quance et al. (46). The soybean and soybean oil demand (as beans and for export, respectively), are obtained in oilmeal equivalents. Demand for soybean oilmeal is not explicitly represented.

### Cotton

Demand for cotton is expressed in terms of cotton lint at national level as 12 million bales. This is equal to the cotton production level in 1978-79 (68). The predicted estimate by Quance et al. (46) for 1985 is only 11 million bales. The higher level was chosen to better approximate the real world situation.

### Hay and silage

The demand for hay and silage originates from the livestock sector. The demand levels are obtained from Quance et al. (46) projections and are summarized at MR level in Table 3.7.

Table 3.7. Demand for hay and silage (Quance et al. (46))

Market region	Legume hay -----million tons-----	Nonlegume hay -----million tons-----	Silage
Northeast	7.47	10.75	24.64
Southeast	0.20	4.05	6.14
Lake states	20.66	6.56	29.83
Corn belt	18.39	13.26	30.31
Delta states	0.48	4.28	2.48
Southern plains	5.54	8.04	7.60
Mountain	11.86	5.98	10.62
North Pacific	7.14	2.51	3.03
South Pacific	5.00	1.71	2.64
TOTAL	76.74	57.14	117.29

### The Commodity Supply Sector

Supply of commodities is simulated by crop production and transportation sectors. Crop production is simulated by rotation activities defined as combinations of different crops. Each crop may be grown under one of three tillage practices - conventional tillage with residue removed, conventional tillage with residue left and reduced tillage. Also, production may take place on dry or irrigated land. Crop yields in each rotation are predicted with Spillman type of production functions estimated by Stoecker (56). Cost of producing crops in each PA under each tillage practice is obtained as explained by Boggess (8).

Demand for water and nitrogen is generated by crop rotations and exogenously specified requirements. Demand for land is generated by crop rotation activities and exogenous agricultural (crops not included in the model) and non-agricultural (urban development) uses. Demand for energy inputs is generated by crop rotations, commercial fertilizer nitrogen supply, irrigation (water supply) and transportation activities. The energy requirement coefficients by type of energy sources were developed for crop rotation activities as explained by Dvoskin and Heady (18). Energy requirement coefficients for transportation, nitrogen and water supply were obtained from Dvoskin and Heady (18).

### The Input Supply Sector

Supply of land and water is defined at PA level.

Amount of land available in each PA is derived from the Conservation Needs Inventory (CNI) county acreages as explained by Boggess (8). The land base by five land classes is shown in Table 3.8 for 1985. The land base was derived from the 1967 CNI by adjusting for projected wetland drainage, irrigation development, and conversions to urban and other nonagricultural uses between 1967 and 1985 (Meister and Nicol (41)).

Table 3.8. Land class and subclass aggregations to the five land quality classes

Land quality class	Inventory class-subclasses <sup>a</sup>	Acres
1	I, II <sub>wa</sub> , III <sub>wa</sub>	64,596,000
2	rest of II, III, IV, all of V	213,385,000
3	III <sub>e</sub>	71,001,000
4	IV <sub>e</sub>	29,886,000
5	VI, VII, VIII	14,340,000

<sup>a</sup>wa indicates that the drainage problem has been eliminated.

Of the total 393.208 million acres available crop land base, 19.293 million acres are specified for use by exogenous crops and summerfallow (Boggess (8)). Land base is defined as irrigated and dry land base.

Water supply is defined as surface water and ground water supply. Ground water is further classified as depletable and rechargeable supply. The entire water sub-sector in the model is constructed as described in Collette (12) and Dvoskin and Heady (18). Allowance is made for conversion of exogenous pasture and hay from irrigated to dry land production. Transfer of water among PA's through natural flows, inter-basin transfers or intra-basin transfer is also allowed.

Energy supplies are defined at MR level. Four types of energy inputs are defined. They are - diesel fuel, electricity, natural gas, and liquid petroleum gas (LPG). Energy supplies are defined at fixed prices.

Various components of the model described above are constructed in the model by defining activities and constraints on these activities. A description of these activities and constraints is presented below.

A mathematical statement of the model is as follows:



$$\begin{aligned}
\text{Max } z = & \sum_m \sum_{\ell_1} d_{\ell_1 m} \cdot q_{\ell_1 m} + \frac{1}{2} \sum_m \sum_{\ell_2} \sum_{\ell_1} D_{\ell_1 \ell_2 m} \cdot q_{\ell_1 m} \cdot q_{\ell_2 m} \\
& - [\sum_i \sum_j \sum_k RC_{ijk} X_{ijk} + \sum_m PN_m NB_m \\
& + \sum_m PN_m NL_m + \sum_n WC_n WB_n + \sum_n WTC_n WT_n \\
& + \sum_t \sum_p TC_{pt} \cdot T_{pt} + \sum_s \sum_m ENC_{ms} \cdot EN_{ms}] \quad (3.11)
\end{aligned}$$

subject to a set of constraints which is described later in this section. A brief discussion of the objective function (Equation 3.11), is presented first. The objective function  $z$  in Equation (3.11) may be interpreted as the sum of producers' and consumers' surplus. This particular interpretation should be qualified by a number of definitions and assumptions so that meaning of this quantity is clearly communicated. In the present study, no attempt is made to use the above interpretation of the objective function. Hence the controversy surrounding the issue will not be taken up here.

An explanation of various notations and symbols used in Equation (3.11) is given below.

- $i = 1, 2, \dots, 105$  for PA's;
- $j = 1, 2, 3$  for tillage practices;
- $k = 1, 2, \dots, 330$  for crop rotations in a PA;
- $m = 1, 2, \dots, 9$  for the nine MR's;

- $p = 1, 2, \dots, 6$  for the six commodities which are transported;
- $t = 1, 2, \dots, 176$  for the transportation routes which are defined;
- $s = 1, 2, 3, 4$  for the four energy inputs;
- $\ell_1, \ell_2 = 1, 2, \dots, 5$  for commodities with linear demand functions;
- $RC_{ijk}$  = cost, in dollars per acre, of crop rotation  $k$  with tillage practice  $j$  in PA  $i$ ;
- $x_{ijk}$  = level of crop rotation activity  $k$ , with  $j^{\text{th}}$  tillage practice in  $i^{\text{th}}$  PA;
- $PN_m$  = price of fertilizer nitrogen, in dollars per pound, in  $m^{\text{th}}$  MR;
- $NB_m$  = level of nitrogen buying activity in  $m^{\text{th}}$  MR;
- $NL_m$  = level of livestock residue expressed as nitrogen fertilizer equivalent in  $m^{\text{th}}$  MR;
- $WC_n$  = price of water, in dollars per acre-foot, in water supply region  $n$ ;
- $WB_n$  = level of water buying activity in water supply region  $n$ ;
- $WTC_n$  = cost, in dollars per acre-foot of water transferred from water supply region  $n$ ;
- $WT_n$  = level of water transferred through natural flow, water exports or interbasin transfer from water supply region  $n$ ;
- $TC_{pt}$  = transportation cost per unit of commodity  $p$  over route  $t$ ;
- $T_{pt}$  = number of units of commodity  $p$  transported over route  $t$ ;
- $ENC_{ms}$  = cost, in dollars per unit of energy input  $s$  in  $m^{\text{th}}$  MR; and
- $EN_{ms}$  = level of energy input  $s$  used in  $m^{\text{th}}$  MR.

Constraints imposed in the model serve two purposes:  
 1) constraints define relationships among various activities from a mathematical viewpoint; and 2) constraints control the resource availabilities and satisfy other predetermined requirements of production, resource transfer or resource use.

#### Constraints at national level

Two constraints are defined at national level.

1. Cotton demand row:

$$\sum_i \sum_j \sum_k y_{4ijk} x_{ijk} \geq 12,000 \quad (3.12)$$

Supply of cotton is constrained to be at least as much as the predetermined demand for cotton (12 million bales of cotton lint).

2. National energy balance row:

$$\sum_m EBCA_m - EBCA_0 \leq 0 \quad (3.13)$$

The national level energy supply ( $EBCA_0$ ) is constrained to at least equal the sum of MR level energy supplies ( $EBCA_m$ ).

#### Constraints at MR level

1. Commodity balance constraints are imposed on all commodities (other than cotton) such that, for the level of commodity demand generated at least an equal amount of demand

is forthcoming.

$$q_{pm} - \sum_{i=1}^{m_i} \sum_j \sum_k y_{pijk} \cdot x_{ijk} + T_{p,m'm} - T_{p,mm'} \leq 0 \quad (3.14)$$

where  $p = 1, 2, 3, 5 \dots 8$  (barley, corn, oats, wheat, soybean oil, sorghum, hay and silage).

$m_i$  = number of PA's in  $i^{\text{th}}$  MR;

$m'$  = MR other than  $m^{\text{th}}$  MR; and  $T_{p,m'm}$  and  $T_{p,mm'}$  are zero for hay and silage.

2. Separable programming constraints define the eigenvalue transformations used to obtain separability in the objective function. The constraints in matrix form are,

$$T'q_m - z = 0 \quad (3.15)$$

where  $T$  is a  $(5 \times 5)$  matrix of eigenvectors of the demand matrix;

$T'$  is the transpose of matrix  $T$ ;

$q_m$  is a  $(5 \times 1)$  vector of quantities in  $m^{\text{th}}$  MR; and

$z$  is a  $(5 \times 1)$  vector of variables which result in separable objective function.

3. Energy balance constraints insure that adequate supply of each energy input and energy in terms of megacalories are available to satisfy the demand for energy generated at MR level. The constraints are of following form:

$$\sum_i \sum_j \sum_k EN_{sijk} x_{ijk} + \sum_{i=1}^{m_i} EN_{si,w} WB_i + EN_{s,n} NB_m$$

$$\sum_p \sum_{m'} EN_{sp,mm'} \cdot T_{p,mm'} \cdot EN_{sm} \leq 0 \quad (3.16)$$

for  $m = 1, 2, \dots, 9$ .

4. Nitrogen balance constraints insure that demand for nitrogen in crop production (endogenous and exogenous) does not exceed the supply of nitrogen from commercial fertilizer and livestock sources.

$$\sum_{i=1}^{m_i} \sum_j \sum_k N_{ijk} \cdot x_{ijk} + \sum_{i=1}^{m_i} N_{exog,i} - (NB_m + NL_m) \leq 0 \quad (3.17)$$

where

$N_{ijk}$  is nitrogen required per acre for  $x_{ijk}$ ; and  
 $N_{exog,i}$  is the exogenous demand for nitrogen in  $i^{th}$  PA.

5. Adoption of reduced tillage or the extent to which land may be brought under reduced tillage is limited by this set of constraints. Limits are obtained as percentages of current land area under reduced tillage. The constraints are of following type:

$$\sum_{i=1}^{m_i} \sum_j x_{ij3} \leq MT_m \quad (3.18)$$

where

$MT_m$  is the maximum land area which can be brought under reduced tillage in  $m^{th}$  MR.

### Constraints at PA level

Land and water supply-demand balances are defined at PA level:

1. Land constraints are specified for dry and irrigated land separately as follows:

$$a. \text{ Dry land: } \sum_j \sum_k x_{ijk,d} \leq DL_i \quad (3.19)$$

$$b. \text{ Irrigated land: } \sum_j \sum_k x_{ijk,r} \leq IL_i \quad (3.20)$$

where subscripts d and r distinguish  $x_{ijk}$  as dry land or irrigated land crop rotations, respectively;

$DL_i$  = dry land available for endogenous crops in  $i^{th}$  PA; and

$IL_i$  = irrigated land available for endogenous crops in  $i^{th}$  PA.

2. Water balance constraints are defined as follows:

$$\begin{aligned} & \sum_k \sum_j W_{ijk} \cdot x_{ijk,r} + W_{i,exog} + WN_i \\ & - (WB_i + WG_i + WD_i + WHAYH_i + WHAYP_i) \\ & - \sum_{i'} WT_{i',i} \leq 0 \end{aligned} \quad (3.21)$$

where

$W_{ijk}$  = water required in acre-foot per acre for  $x_{ijk,r}$  in  $i^{th}$  PA;

$W_{i,exog}$  = exogenous demand for water in  $i^{th}$  PA;

$WN_i$  = water transferred from  $i^{th}$  PA to other PA's;

$WB_i$  = surface water supplied in  $i^{th}$  PA;

$WG_i$  = rechargeable ground water supplied in  $i^{th}$  PA;

$WD_i$  = depletable ground water supplied in  $i^{th}$  PA;

$WHAYH_i, WHAYP_i$  = water obtained from conversion of exogenous hay and pasture from irrigated to dry land production; and

$WT_{i,i'}$  = water transferred from other ( $i'$ ) PA's to  $i^{th}$  PA.

Bounds on activities used in the programming model are also essentially constraints. Upper bounds of one on the linear segments of nonlinear variables were used. Energy supply restrictions used in the model were also achieved by putting upper bounds on energy supply activities. Bounds on water supply and water transfer activities were used as explained by Collette (12).

#### A Comparison of the Present Model with Models Employed in Some Previous Studies

In two previous studies, employing a cost minimizing linear programming models, Dvoskin and Heady (18) and Dvoskin, Heady and English (19) analyzed the impact of alternative energy policies and energy situations on U.S. agriculture. The present study employs a separable programming model with a quadratic objective function for similar purpose. In several respects the two approaches have differences. In this section only two major differences reflecting the flexibility in the present model with respect

to changes in output level are discussed.

The commodity demand, at regional or national level, in the linear programming models is held fixed at pre-determined levels for each commodity. Although existence of transportation network would allow changes in output levels through interregional shifts in crop production, the fixed demand levels are not consistent with changes in output prices. As input prices increase, output prices also are expected to increase.

In Figure 3.4, horizontal and vertical axes show the amounts of inputs A and B, respectively, used in the production of output Q. Let  $X_1$  and  $X_2$  be two processes by which output Q can be produced. Let negative of slope of price line  $P_1$  equal the ratio of price of A to initial price of B. The optimal, or least cost, combination of inputs for output level  $Q_1$  is  $A_1$  units of input A and  $B_1$  units of input B. Let the price of input B now increase such that the new price line, whose slope equals negative of the ratio of price of A to increased price of B, be  $P_2'$ . The optimal input levels for the new price line are  $B_2$  units of input B and  $A_2$  units of input A, if output level remains at  $Q_1$ . In interregional competition linear programming models, output levels of commodities may change at regional levels, but total output at national level remains constant. If total output is permitted to change a reduction



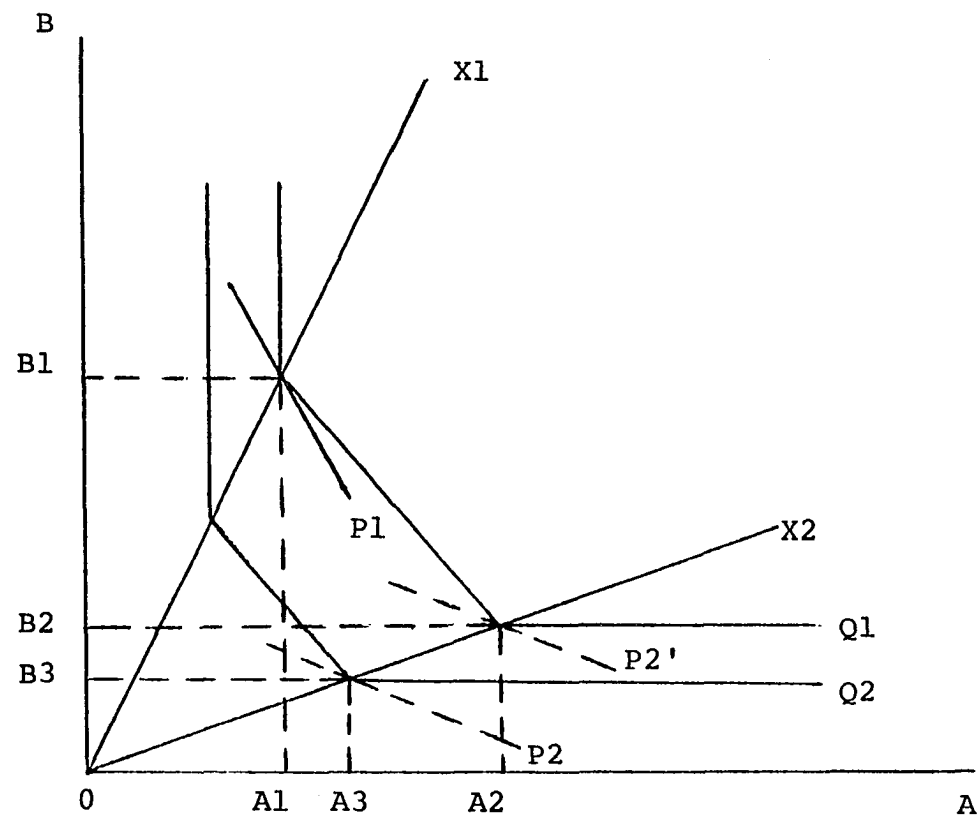


Figure 3.4. Illustration of substitution effect and output effect of a change in input price

to say  $Q_2$ , would result in optimal input levels  $B_3$  ( $< B_2$ ) units of input B and  $A_3$  ( $< A_2$ ) units of input A. In firm theory, changes in input use from  $B_1$  to  $B_2$  and  $A_1$  to  $A_2$  are termed "substitution effect" and changes from  $B_2$  to  $B_3$  and  $A_2$  to  $A_3$  are termed "output effect". Thus, input adjustments required are greater in a model which permits output level to vary than a model in which output level is held constant.

The linear programming models thus have less ability to reflect all the resource adjustments as compared to programming models which do not hold demand for commodities at predetermined levels.

In another respect the present model differs from previous models developed at CARD. Livestock feed demand for grains is specified in corn equivalent in the present model. The feed grains (barley, corn, oats, sorghum) and wheat can substitute for one another in meeting feed demand. In models with exogenous livestock sector, such as the models employed by Dvoskin and Heady (18) and Dvoskin, Heady and English (19) feed demand is specified as demand for individual grain crops. The possibility of substitution among grain crops results in the composition of feed supply (and demand) which is least expensive at prices obtained in the model.

### Alternative Energy Situations Analyzed

The present study employs a programming model of U.S. agriculture to analyze the impacts of changes in energy prices and energy supplies on a number of variables related to agricultural sector. Changes occurring in energy situation have been mainly, increasing energy prices and continued growth in demand in the face of declining fossil fuel reserves. Expectations with respect to energy prices are for the prices to remain at high levels with increasing trends due to continued high demand for oil throughout the world and prospects of oil price decontrol in the United States. More specific predictions with respect to future energy prices are difficult to make. Oil pricing by OPEC countries has become less predictable (55). However, in order to study the impact of energy price increases two alternative price situations were selected in this study.

Table 3.9 summarizes prices of gasoline, diesel, LPG, electricity and natural gas under different assumptions. As natural gas price is to be decontrolled gradually, and electric utility rates continue to be regulated, increases in natural gas and electricity prices may be expected to be more gradual than increases in the prices of gasoline, diesel or LPG. With this viewpoint, two alternative price levels were chosen to characterize two different energy

Table 3.9. Energy prices under alternative assumptions (22, 67)

Energy source	\$ per	Prices in 1975 dollars				Prices in 1979 dollars (CPI=223.7 (1967=100))				Prices in 1985 dollars (7% inflation rate over 1979)			
		Actual	No. of times			Actual	No. of times			Actual	No. of times		
			2	4	6		2	4	6		2	4	6
		1975	1975	1975	1975	1979 <sup>c</sup>	1975	1975	1975	1979	1975	1975	1975
Gas <sup>a</sup>	gallon	0.498	0.996	1.992	2.988	0.91	1.38	2.77	4.15	1.28	1.94	3.89	5.82
Diesel	gallon	0.391	0.782	1.564	2.346	0.82	1.09	2.17	3.26	1.15	1.53	3.04	4.57
LPG	gallon	0.304	0.608	1.216	1.824	0.48	0.85	1.69	2.54	0.67	1.19	2.37	3.56
Elect.	kwh	0.0307	0.0614	0.1228	0.184	0.06	0.09	0.17	0.26	0.08	0.13	0.24	0.37
N. gas <sup>b</sup>	Thous. cu. ft.	0.7603	1.5206	3.0412	4.562	2.14	2.11	4.23	6.34	3.00	2.96	5.93	8.89

<sup>a</sup>Regular, bulk delivery rates.

<sup>b</sup>Price to industrial users.

<sup>c</sup>Average of August, September, and October prices.

price situations:

1. low energy price scenario; and
2. high energy price scenario.

The selected price levels under the above two price scenarios are presented in Table 3.10.

Uncertainties exist with respect to future energy supplies. Conservation of energy, by reducing the use of energy or by more efficient use of energy is being recognized as one of the short- and medium-term measures to deal with the energy crisis, by researchers and policy makers. Energy supplies to agricultural sector (including manufacture of fertilizers and pesticides) are restricted to 90 percent

Table 3.10. Selected prices (in 1975 and 1985<sup>a</sup> dollars)

Energy source	\$ per	Low energy prices for 1985		High energy prices for 1985	
		in		in	
		1975 \$	1985 \$	1975 \$	1985 \$
Gasoline	gallon	1.49	2.91	2.988	5.83
Diesel	gallon	1.17	2.28	2.346	4.58
LPG	gallon	0.608	1.19	1.824	3.56
Elect.	Kwh	0.06	0.12	0.12	0.23
N. gas	Thous. cu. ft.	2.28	4.45	3.04	5.93

<sup>a</sup>To illustrate the price increases, the 1985 prices are also derived in current dollars assuming a 7% inflation rate annually starting 1979.

of the energy used under the low energy price scenario, in two alternatives analyzed in the present study.

The two energy situations related to alternative energy supplies are: 1) energy supply is restricted on a national basis; and 2) energy supply is restricted on a regional (MR) basis. Restricting energy supply at national level will result in efficient allocation of available energy in crop production, transportation and manufacture of agricultural inputs. But in the short-run, energy supply reductions (or conservation) may be expected to result on a more uniform basis at a disaggregated level (such as across states). Hence, the alternative with regional level restrictions on energy supplies is also analyzed in the present study. The matrix indicating the status of variables (or instruments of change) in the study is summarized in Table 3.11.

Table 3.11. The alternative energy situations analyzed

Alternative	Energy prices	Energy use	Exports
Base run or low energy price scenario	Low	Unrestricted	OBERS high
High energy price scenario	High	"	"
National energy supply reduction	Low	10% reduction at natl. level relative to base run	"
Regional energy supply reduction	Low	10% reduction at regional (MR) level relative to base run	"

## CHAPTER IV. RESULTS AND DISCUSSION

The impacts of alternative energy futures simulated in the model, on different aspects of agriculture are evaluated, using results of the present analysis. The current and expected changes in energy situation may be expected to result in several changes in the agricultural sector. These changes may lead to efficient allocation of resources under the altered economic conditions within the industry. This study reflects only some of the possible changes in the agricultural sector.

In the present chapter, results are presented at national and MR level on resource use in crop production and imputed prices of resources; production of farm commodities and their prices; crop production costs, food costs and net farm income. To recapitulate from the previous chapter, the four energy situations considered in the study are: 1) low energy price scenario; 2) high energy price scenario; 3) energy supply reduction at national level; and 4) energy supply reduction at MR level. For comparison of results, the low energy price scenario is selected as the base run solution or a reference solution.

Before presenting the results, a brief review of some of the economic concepts used in summarizing the results of the study may be useful.

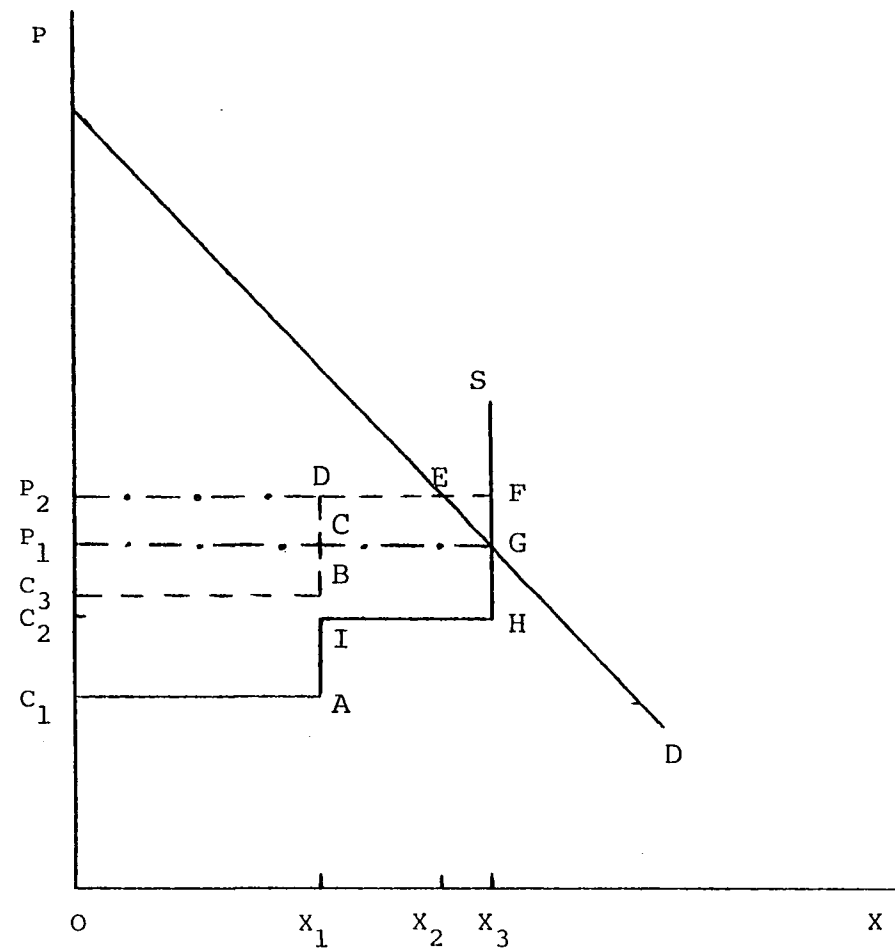


Figure 4.1. Illustration of changes in rental price of inputs



Figure 4.1 illustrates a supply and demand situation in an input market. Let the horizontal axis  $OX$  represent quantity of input  $x$  and the vertical axis  $OP$  represent price of input  $x$ . Supply curve for the input is a stepped linear function represented by  $C_1AIHS$ . The downward sloping linear function  $DD$  represents the input demand function. The situation illustrated in Figure 4.1 is only one of many possible alternative situations. Under competitive conditions, the equilibrium quantity of input and price are  $OX_3$  and  $OP_1$ , respectively, in the above figure. Economic rent is defined as the return to a resource due to fixed level of its supply in the short-run, or as any return above the amount necessary to retain the factor in production (29). Under competition, resource holders will cover cost of supplying the input up to  $OX_1$  if input price is  $OC_1$ . Similarly, at price  $OC_2$ , resource holders supply additional  $X_1X_3$  units of the input  $x$ . But market price is determined at  $OP_1$ . Hence, the resource holders get a rental price of  $C_1P_1$  per unit up to  $OX_1$  and a rental price  $C_2P_1$  for  $X_1X_3$  of input  $x$ . The price charged to resource users is  $OP_1$  and net income to resource holders is the sum of two rectangles -  $C_1ACP_1$  and  $IHGC$ .

Now suppose, supply function for the input shifts from  $C_1AIHS$  to  $C_3BDFS$ . For example, the input under consideration may be water. Let  $OX_1$  be the amount of surface water

available in a PA and  $X_1X_3$  be the amount of ground water available in a PA. As energy prices increase, cost of supplying water also increases. But, shift in supply price of surface water and ground water may not occur at the same rate. Supply price of ground water may increase more than the increase in surface water. The difference in the rate of increase in input supply price by source of supply, gives rise to difference in rental prices for the same input. Changes in input prices and rental prices may further be influenced by shifts in the demand for input. Changes in output prices will shift the demand schedule for inputs - i.e., for a given input price, a smaller (higher) level of input is demanded with a downward (upward) shift in the demand for the input.

A related situation occurs when supply of an input is restricted to lower level than would be used under a given price and production condition. In the present study, restriction on energy use (or supply) represents this situation. The scarce (energy) input in such a case earns a rental price.

#### Energy Use and Energy Prices

In the United States, technology and price conditions have favored a capital intensive and energy intensive agriculture. Researchers (for example, Berry (7)) have pointed out that

because rate of increase in energy expenses is smaller relative to increases in crop prices, impact of increases in energy prices would be minimal in crop production. Such conclusions also suggest that with inelastic demand for energy in crop production, energy supply shortages rather than increases in energy prices will have greater impact on energy use in agriculture.

Energy supplied by diesel fuel, electricity, natural gas and LPG is required for crop production (for operations from land preparation to crop drying), irrigation, inter-regional transportation of commodities and finally for the manufacture of agricultural inputs. The total use of energy, by type of source, under different energy alternatives is reported in Table 4.1. Under all four energy situations, the order of importance of source of demand for energy inputs remains unchanged. The major use of diesel occurs in field operations (90 percent of diesel used in the base run is used for field operations) followed by irrigation and transportation. Irrigation and manufacture of potassium (K) and phosphorous (P) fertilizers are the major sources of demand for electricity, followed by manufacture of nitrogen. Manufacture of nitrogen requires largest proportion (92 percent in the base run) of total natural gas used in agriculture. Manufacture of P and K fertilizers and irrigation account for the remaining uses of

Table 4.1. Use of energy inputs in agriculture at national level<sup>a,b</sup>

	Low energy prices	High energy prices		National energy supply reduction		Regional energy supply reduction
<u>Diesel (mill. gal.)</u>						
Crop production	4116.33	4041.72	(-1.81)	3831.03	(-6.93)	3700.22 (-10.11)
Irrigation	422.18	407.43	(-3.49)	429.74	(+1.79)	564.90 (+33.81)
Transportation	32.07	20.50	(-36.08)	12.78	(-60.15)	15.93 (-50.33)
TOTAL	4570.58	4469.65	(-2.21)	4273.55	(-6.50)	4281.05 (-6.34)
<u>Electricity (mill. kwh)</u>						
Nitrogen	876.11	872.89	(-0.37)	841.42	(-3.96)	850.32 (-2.94)
P & K fertilizers	3124.03	3179.07	(+1.76)	3173.59	(+1.56)	3160.67 (+1.17)
Irrigation	3686.71	3294.61	(-10.64)	2552.78	(-30.76)	2621.88 (-28.88)
TOTAL	7686.85	7346.51	(-4.43)	6451.30	(-16.07)	6632.87 (-13.71)
<u>Natural gas (bill. cu. ft.)</u>						
Nitrogen	327.798	326.50	(-0.40)	314.81	(-3.96)	318.15 (-2.94)
P & K fertilizers	18.880	18.18	(-3.71)	17.92	(-5.07)	19.10 (+1.19)
Irrigation	11.308	9.31	(-17.71)	6.05	(-46.50)	5.34 (-52.80)
TOTAL	357.986	354.99	(-1.09)	338.78	(-5.37)	342.59 (-4.30)
<u>LPG (mill. gal.)</u>						
Irrigation	24.61	10.72	(-56.55)	11.47	(-53.39)	9.28 (-62.29)
Crop drying	563.88	533.05	(-5.47)	545.46	(-3.27)	493.58 (-12.47)
TOTAL	588.49	543.77	(-7.60)	556.93	(-5.36)	502.86 (-14.55)

<sup>a</sup>The energy used by exogenous crops for irrigation and nitrogen manufacture is included in the figures shown in this table.

<sup>b</sup>Numbers in parentheses indicate percentage change over the low energy price scenario.

natural gas. Crop drying accounts for over 90 percent of total LPG used under all four energy alternatives. Under higher energy prices, as suggested by the economic theory of firm, use of all energy inputs is reduced, as compared to the amount of energy inputs used at lower energy prices. But the decline is not uniform. Use of LPG decreases more than any other energy input. In addition to the fact that energy prices were increased at different rates, energy content of various energy inputs is different. This accounts for a disuniform change in the use of inputs due to changes in energy situation. Also, a change in the crop mix which maximizes consumers' and producers' welfare under higher energy prices will result in altered combination of energy inputs compared to base run. With respect to changes in energy inputs by source of demand, greater disuniformity is observed. For example, electricity used in the manufacture of P and K fertilizers actually increases with increase in energy prices while natural gas required for P and K manufacture decreases. Proportionately, P fertilizers require larger inputs of electricity relative to natural gas, as compared to K fertilizers. Hence, a change in the crop mix, from one with relatively higher potash needs to relatively higher phosphorous needs accounts for the increased electricity use and decreased natural gas use in the manufacture of P and K fertilizers as energy prices increase.

Table 4.2. Use of energy to manufacture pesticides

	Low energy prices	High energy prices	National energy supply reduction	Regional energy supply reduction
	----- (million mcals) -----			
Northeast	133.51	286.17	289.33	307.22
Southeast	163.06	164.02	143.22	159.45
Lake states	942.56	982.25	1003.35	936.13
Corn belt	1730.09	1720.19	1759.80	1805.75
Delta states	790.93	910.47	951.79	948.56
Southern plains	1149.99	1162.46	1138.79	1171.40
Mountain	230.82	206.15	200.56	207.22
North Pacific	146.36	135.69	134.35	119.25
South Pacific	56.27	56.85	18.40	28.05
U.S. TOTAL	5343.59	5624.25	5639.59	5683.03

As shown in Table 4.2, energy used for manufacture of pesticides also increases, when energy prices increase. Under minimum tillage diesel fuel requirements are lower as compared to conventional tillage, but pesticide application rates are higher. As energy prices are increased, crop acreage under minimum tillage increases from 49 million acres under low energy price scenario to 56 million acres. Thus, more energy is required for manufacture of higher levels of pesticides used.

With restrictions on energy supplies, at national and regional levels, total use of all energy inputs is reduced. In general, pattern of reduction in energy use is similar to the pattern when energy prices increase, including the increase in electricity used for P and K fertilizers, and the increase in energy for pesticides as shown in Tables 4.1 and 4.2. Crop acreage under minimum tillage increased from 49 million acres under low energy price scenario to 69 and 73 million acres under energy supply restrictions at national and MR levels, respectively. Energy used for pesticides accordingly increases under latter two alternatives.

For irrigation, a shift away from LPG, electricity and natural gas and toward diesel as fuel source is observed. Comparison between impact of energy reduction at national and regional levels shows that consumption of diesel,

electricity and natural gas is slightly higher with regional level energy supply reductions. The only reduction is observed in the use of LPG. The relatively high content of energy in LPG would reduce the total energy use through reductions in LPG more than through reductions in other energy inputs.

Thus, in general, reductions in the use of energy inputs due to higher energy prices and supply restrictions on energy may be accomplished in one or more of the following ways:

- a. reductions in output levels of some commodities;
- b. shift from more energy-intensive inputs to less energy-intensive inputs;
- c. shift in crop production from PA's where more energy inputs are required to PA's where energy requirements are less; and
- d. shift from more energy-intensive methods of production to less energy-intensive methods of production.

Results on energy use point in the same direction as the results of studies by Dvoskin and Heady (18). The changes in the use of energy inputs at MR level are reported in Tables 4.3 through 4.6. Use of energy inputs is more uniformly distributed across MR's with energy supply restricted at regional level rather than energy supply restricted at national level.

By restricting energy supplies, shadow prices for



Table 4.3. Total use of diesel in agriculture, by market region, under alternative energy situations

Market region	Low energy prices	High energy prices	National energy supply reduction	Regional energy supply reduction
	-----million gallons-----			
Northeast	317.555	323.559	332.046	349.681
Southeast	196.467	197.845	174.766	183.548
Lake states	914.049	883.918	881.359	826.282
Corn belt	1696.659	1675.434	1645.375	1602.233
Delta states	310.937	312.042	294.753	282.270
Southern plains	577.844	556.659	451.233	539.288
Mountain	382.322	347.157	344.971	351.480
North Pacific	113.727	111.423	110.912	103.162
South Pacific	61.015	61.611	28.136	43.110
U.S. TOTAL	4570.575	4469.647	4273.55	4281.053

Table 4.4. Total use of electricity in agriculture, by market region, under alternative energy situations

	Low energy prices	High energy prices	National energy supply reduction	Regional energy supply reduction
	-----million gallons-----			
Northeast	357.857	392.652	409.112	421.601
Southeast	267.729	272.677	228.077	243.811
Lake states	835.757	824.897	823.806	778.067
Corn belt	1513.993	1406.796	1373.317	1359.973
Delta states	317.301	322.449	309.798	296.759
Southern plains	838.108	775.091	589.921	674.771
Mountain	840.154	801.964	784.949	803.655
North Pacific	820.482	697.582	697.966	599.478
South Pacific	1895.471	1852.461	1234.356	1454.757
U.S. TOTAL	7686.852	7346.57	6451.300	6632.871

Table 4.5. Total use of natural gas in agriculture, by market region, under alternative energy situations

Market region	Low energy prices	High energy prices	National energy supply reduction	Regional energy supply reduction
	-----billion cubic ft.-----			
Northeast	22.227	23.088	27.271	30.884
Southeast	21.766	22.395	16.824	18.475
Lake states	52.157	51.171	51.290	45.512
Corn belt	98.968	98.093	103.678	96.855
Delta states	29.376	29.694	27.872	26.086
Southern plains	73.135	70.411	54.420	65.171
Mountain	18.285	17.842	16.308	16.252
North Pacific	26.571	25.745	25.671	24.387
South Pacific	15.501	15.636	15.449	18.969
U.S. TOTAL	357.986	354.074	338.782	342.592

Table 4.6. Total use of LPG in agriculture by market region, under alternative energy situations

Market region	Low energy prices	High energy prices	National energy supply reduction	Regional energy supply reduction
	-----million gallons-----			
Northeast	78.584	60.217	69.696	78.209
Southeast	0	0	0	0
Lake states	208.749	204.410	204.768	183.992
Corn belt	274.002	253.858	266.277	224.718
Delta states	4.166	4.042	3.585	2.745
Southern plains	13.830	11.937	4.568	4.758
Mountain	8.762	8.641	7.852	8.061
North Pacific	0.160	0.160	0.160	0.149
South Pacific	0.222	0.223	0.027	0.224
U.S. TOTAL	588.476	543.488	556.932	502.855

energy can be obtained in the programming model. These shadow prices are essentially, the rental prices for energy. The rental prices are returns to resources over and above the price actually paid (the supply price) as production cost. In Table 4.7 energy's rental prices are summarized for the energy alternatives with restricted supplies. The rental price reported is the value of an addition megacalorie of energy supplied to agriculture.

When energy supplies are unrestricted, value of energy is given by the price of energy inputs as specified exogenously to the model. Under energy supply reduction at national level, imputed rental price is uniform over all MR's at approximately 8 cents per megacalorie. This imputed rental price is substantially higher than the price (reported as shadow price) obtained by Dvoskin and Heady (18) as about 4 cents per megacalorie. The average prices of energy inputs used by Dvoskin and Heady (18) were higher than the average prices used in the present study. For example, the average prices for diesel were 1.37 dollars per gallon and 94 cents per gallon in the two studies, respectively. Hence the difference in net impacts of restricting energy supplies at national level on the value of energy in the margin would be small in the two studies.

When energy supplies are restricted at regional level,

Table 4.7. Energy shadow price, by market region

	National energy supply reduction -----dollars/Mcal-----	Regional energy supply reduction -----
Northeast	0.08340	0.07575
Southeast	0.08340	0.14056
Lake states	0.08340	0.15929
Corn belt	0.08340	0.14548
Delta states	0.08340	0.15327
Southern plains	0.08340	0.10749
Mountain	0.08340	0.13215
North pacific	0.08340	0.24054
South Pacific	0.08340	0.06403
U.S. AVERAGE	0.0834	0.13906

energy rental prices reveal the relative importance of additional supply of energy to each MR. The level of rental price is a function of price of the commodity produced and the marginal productivity of the input. Highest energy rental price is obtained in the North Pacific region. The lake states, delta states and corn belt follow in the level of rental price for energy after North Pacific. The prices are higher when supplies are restricted at MR level than when supplies are restricted at national level in all

MR's except in Northeast and South Pacific.

In summary, increases in energy price cause overall reduction in energy use in agriculture by about 2.64 percent (measured in megacalories). Energy use in field operations (crop production) and irrigation are affected substantially due to higher energy prices. A shift in production method, from conventional to minimum tillage is observed as an energy saving change under higher energy prices as well as lower energy supplies. Changes in energy use at MR level follow the pattern at national level. The results confirm the view that increased acreage under minimum tillage and reduced energy use for irrigation would be the changes which may result due to the changing energy situation.

Land, Water and Nitrogen: Use for Endogenous  
Crops and Imputed Prices

The quantity of input used in production is a function of productivity of the input, its price relative to output price and other input prices, and level of production. The extent of substitution of one input for another input is limited by model specification. For example, input requirements of nitrogen and water per acre are fixed at given levels for a crop in a given PA. But the requirements differ from one PA to another, and from one crop to

another. Hence, shift in the production of a crop from PA's currently producing the crop to PA's which currently do not produce the crop, or a change in crop-mix in a PA change the total quantities of water and nitrogen used when energy prices are changed relative to other input prices. Other means of achieving input substitutions in the model are change from irrigated land to dry land farming and change from conventional tillage to minimum tillage. As the primary instruments of change in the alternative situations under study are prices and supply of energy, the energy requirements of each input affect the quantity of its use.

Total land use by MR, by dryland and irrigated land under the four alternative energy situations is summarized in Table 4.7. The average rental prices of land per acre at MR level and weighted average prices at national level are also reported in Table 4.7. From a total of 375.19 million acres available for endogenous crops, more than 93.6 percent is brought under crops in all four energy scenarios. Largest acreage (359.14 million acres) is used in the base run. Reductions in land use as energy prices increase or supplies decrease occur mainly due to reduction in crop output. For example, if output levels were held constant for all crops, acreage may increase reflecting substitution of land for energy. The substitution of land



for energy may be accomplished by adopting low yield and low energy consuming methods of production. Although substitution of land for energy is not evidenced by increase in land used, substitution of dryland for irrigated land is evident. Use of dryland increases slightly, by 0.46 million acres, but irrigated land use decreases by 1.52 million acres when energy prices are increased as compared to base run. When energy supplies are reduced at national and regional levels, irrigated land use decreases substantially as compared to base run. The higher energy requirements for irrigated crop production make dryland production more attractive (because of the energy savings achieved) as compared to irrigated farming, as energy prices increase and energy supplies are reduced. This aspect may appear to be contrary to the trend in irrigated farming in the United States. Sloggett (51) for example, notes that, "acreage irrigated from ground water increased by 4.5 million acres, with Nebraska and Kansas accounting for 2.4 million acres" during the year 1978. Ground water requires even more energy for irrigation than does surface water. Thus, impact of higher energy prices has not yet been to reduce irrigated acreage under crops. Sloggett (51) suggests that, "as energy is only a small part of production expense, especially in great plains, a small increase in output price would offset the increases in energy prices."

Table 4.8. Land use and imputed shadow prices for land, by market region, under alternative energy situations

	Low energy prices		High energy prices		National energy supply reduction		Regional energy supply reduction	
	acres/ mill	\$/ acre	acres/ mill	\$/ acre	acres/ mill.	\$/acre	acres/ mill.	\$/acre
<u>DRY LAND</u>								
Northeast	22.85	80.24	22.85	81.46	22.85	124.39	22.85	246.72
Southeast	17.79	26.80	17.93	29.02	16.67	38.26	17.30	42.09
Lake states	69.80	35.23	69.80	35.32	69.80	44.24	68.37	43.29
Corn belt	105.75	26.14	105.75	69.34	105.75	89.59	105.75	106.88
Delta states	26.14	23.90	26.14	26.73	25.47	15.10	24.68	26.44
Southern plains	51.20	28.05	49.82	32.03	46.50	28.80	53.63	61.21
Mountain	32.59	33.18	32.59	29.37	32.87	48.63	31.44	80.23
North Pacific	11.36	36.80	11.36	46.33	11.36	57.76	11.21	7.82
South Pacific	1.62	142.60	1.70	134.68	1.69	154.22	2.22	101.24
U.S. TOTAL	337.48	45.75	337.94	47.90	332.95	60.91	237.38	87.11
<u>IRRIGATED LAND</u>								
Northeast	0	0	0	0	0	0	0	0
Southeast	0	0	0	0	0	0	0	0
Lake states	0.13	56.66	0.13	76.22	0.13	88.46	0.13	102.37
Corn belt	4.33	50.85	3.85	43.76	3.34	49.40	3.31	63.66
Delta states	0	0	0	0	0	0	0	0
Southern plains	3.23	9.25	2.59	3.22	1.30	1.04	2.15	4.00
Mountain	5.44	35.42	5.21	32.17	4.84	38.74	5.11	60.24
North Pacific	4.30	81.58	4.36	85.30	4.36	123.60	4.13	80.81
South Pacific	4.24	53.93	4.24	48.38	4.21	53.49	4.84	89.06
U.S. TOTAL	21.66	47.53	20.24	46.01	18.18	62.12	19.67	66.38

As the present study yields normative results or the "optimal allocation of inputs", all the results may not coincide with the real world. However, our results may be considered as long-term impacts. Thus, reduction in irrigated land use may be considered as a long-term impact of the energy crisis, unless improvements in efficiency of energy use actually reduce the cost of production associated with energy.

The interregional changes in land use also are summarized in Table 4.8. Rental prices of dry and irrigated land are highest when energy supplies are restricted at regional levels. Rental prices for inputs including land depend upon productivity of inputs, cost of production associated with the inputs and output price. Therefore, as energy prices increase, cost of production on irrigated land is greater than on dryland. Although productivity measured as yield per acre is greater on irrigated land, the higher production costs result in reduced land rental price for irrigated land with increases in energy prices. For dryland, rental values increase as energy prices increase.

Measured at market region level, the rental prices for land follow national pattern in most cases. In some cases such as in the case of North Pacific region, for both dryland and irrigated land, rental prices decline as energy prices increase. Increase in output prices in such

cases is less than increased cost of energy. But at national level, rental prices for land, in general, increase. Regional differences in changes in the rental prices for land arise, basically due to the same reasons as those for the differences in changes in dry land and irrigated land rental prices.

Land use by crops, without distinction between dry and irrigated land, at national and regional levels is summarized in Tables 4.9 through 4.12 for feed grains (barley, corn, sorghum and oats), wheat, soybean and roughages (legume and nonlegume hay, and corn and sorghum silage). The feed grain acreage of 118.75 million acres in the base run is slightly larger than the estimated 104.3 million acres under feed grains in 1978-79 (68). There are no major interregional shifts in cropping patterns due to changes in energy situation. Lake states and corn belt remain the major producers of feed grains accounting for over 53 percent of total U.S. feed grain acreage in the base run. Lake states and corn belt also remain major producers of wheat, followed by mountain region and North Pacific region. Comparing with the USDA (68) estimate of 56.1 million harvested acres of wheat in 1978-79, the base run result of 69.66 million acres under wheat appear realistic. In the case of soybean, the corn belt accounts for over 61 percent of U.S. soybean acreage in all four

Table 4.9. Feed grain acreage, by market regions, under alternative energy situations

	Low energy prices		High energy prices		National energy supply reduction		Regional energy supply reduction	
	Dry land	Irrigated land	Dry land	Irrigated land	Dry land	Irrigated land	Dry land	Irrigated land
	-----million acres-----							
Northeast	7.90	0	8.95	0	10.49	0	11.68	0
Southeast	9.08	0	9.30	0	8.14	0	8.62	0
Lake states	29.37	0.13	28.89	0.13	29.37	0.13	29.61	0.13
Corn belt	33.50	2.14	35.55	1.67	34.37	1.16	34.87	1.13
Delta states	11.35	0	11.39	0	11.18	0	10.60	0
Southern plains	14.42	3.14	14.01	2.50	9.36	1.26	15.55	2.10
Mountain	9.42	1.79	9.51	1.79	9.41	1.39	9.43	0.77
North Pacific	2.93	0	3.24	0	3.23	0	3.13	0
South Pacific	0.80	1.02	0.80	1.02	0.87	1.03	1.80	1.20
U.S. TOTAL	118.75	8.23	121.64	7.12	116.43	4.97	125.29	5.33

Table 4.10. Acreage under wheat, by market regions, under alternative energy situations

	<u>Low energy prices</u>		<u>High energy prices</u>		<u>National energy supply reduction</u>		<u>Regional energy supply reduction</u>	
	<u>Dry land</u>	<u>Irrigated land</u>	<u>Dry land</u>	<u>Irrigated land</u>	<u>Dry land</u>	<u>Irrigated land</u>	<u>Dry land</u>	<u>Irrigated land</u>
	-----million acres-----							
Northeast	1.13	0	2.98	0	2.66	0	2.25	0
Southeast	0.54	0	0.58	0	0.18	0	0.34	0
Lake states	14.17	0	14.62	0	14.58	0	13.66	0
Corn belt	17.85	0	17.71	0	18.26	0	17.56	0
Delta states	2.74	0	2.71	0	2.57	0	2.45	0
Southern plains	18.13	0	19.31	0	20.01	0	21.21	0
Mountain	8.94	2.85	8.63	2.78	10.11	2.53	9.77	2.90
North Pacific	4.15	3.97	3.59	4.13	2.53	4.13	2.97	4.13
South Pacific	0	1.33	0.04	1.33	0.03	1.33	0	1.57
U.S. TOTAL	69.66	8.15	70.15	8.24	70.92	7.99	70.22	8.60

Table 4.11. Acreage under roughages, by market regions, under alternative energy situations

	Low energy prices		High energy prices		National energy supply reduction		Regional energy supply reduction	
	Dry land	Irrigated land	Dry land	Irrigated land	Dry land	Irrigated land	Dry land	Irrigated land
	-----million acres-----							
Northeast	8.73	0	8.76	0	8.86	0	8.83	0
Southeast	2.15	0	2.12	0	2.41	0	2.41	0
Lake states	19.30	0	19.30	0	19.29	0	18.83	0
Corn belt	15.75	0	14.01	0	14.42	0.23	15.47	0.08
Delta states	2.48	0	2.47	0	2.47	0	2.50	0
Southern plains	7.76	0.04	7.66	0.04	8.05	0	7.97	0
Mountain	12.37	0.79	12.61	0.73	12.43	0.67	12.07	1.01
North plains	4.21	0.33	4.54	0.22	4.52	0.22	4.64	0.0
South plains	0.81	1.88	0.87	2.05	0.86	1.85	0.80	2.07
U.S. TOTAL	73.56	3.04	72.33	3.05	73.31	2.97	73.53	3.16

Table 4.12. Acreage under soybean, by market regions, under alternative energy situations

	Low energy prices		High energy prices		National energy supply reduction		Regional energy supply reduction	
	Dry land	Irrigated land	Dry land	Irrigated land	Dry land	Irrigated land	Dry land	Irrigated land
	-----million acres-----							
Northeast	3.09	0	2.16	0	0.85	0	0.09	0
Southeast	5.94	0	5.94	0	5.90	0	5.90	0
Lake states	6.96	0	6.99	0	6.56	0	6.27	0
Corn belt	39.13	2.18	37.90	2.18	37.78	0.18	38.87	0.18
Delta states	7.38	0	7.38	0	7.07	0	6.95	0
Southern plains	0.05	0	0.05	0	0.29	0	0	0
Mountain	1.84	0	1.84	0	1.84	0	1.84	0
North Pacific	0	0	0	0	0	0	0	0
South Pacific	0	0	0	0	0	0	0	0
U.S. TOTAL	64.39	2.18	62.25	2.18	60.28	0.18	59.91	0.18



energy alternatives. Delta states, lake states and south-east are the other major soybean producing market regions. Soybean acreage in the base run at 64.39 million acres, also compares favorably with the USDA (68) estimate of soybean's harvested 63.0 million acres in 1978-79. In the case of cotton, production is concentrated in the northeastern parts of southern plains region and in Delta states. All cotton is produced on dryland. National cotton acreage ranges from 11.04 million acres in low energy price scenario to 11.60 million acres under national level energy supply reduction.

Distribution of the unused or slack land under alternative energy situations is reported in Table 4.13. Largest proportion of unused land is the southern plains region under all energy alternatives. Major portion of slack land is the irrigated land. Higher levels of energy required for irrigated farming would make irrigated farming less profitable as compared to dryland farming, as energy prices increase or supplies reduce. When energy supplies are restricted at MR level, to 90 percent of energy used in the base run, distribution of slack land is wider than under national level energy supply shortage. Overall, unused land increases as energy situation changes from one of lower prices and unrestricted supplies to higher prices or reduced supplies.

Table 4.13. Distribution of unused land by market region, under alternative energy situations

	<u>Low energy prices</u>		<u>High energy prices</u>		<u>National energy supply reduction</u>		<u>Regional energy supply reduction</u>	
	Dry land	Irrigated land	Dry land	Irrigated land	Dry land	Irrigated land	Dry land	Irrigated land
	-----million acres-----							
Northeast	0	0	0	0	0	0	0	0
Southeast	1.23	0	1.09	0	2.35	0	1.72	0
Lake states	0	0	0	0	0	0	1.43	0
Corn belt	0	0	0	0.48	0	0.99	0	1.02
Delta states	0	0	0	0	0.67	0	1.47	0
Southern plains	2.81	7.18	4.09	7.82	7.51	9.11	0.36	8.26
Mountain	0.29	1.02	0.29	1.24	0	1.61	1.43	1.35
North Pacific	0	0.14	0	0.08	0	0.08	0.24	0.31
South Pacific	0.82	0.95	0.74	0.95	0.76	9.77	0.23	0.35
U.S. TOTAL	14.44		16.88		24.06		18.16	

Distribution of crop acreage under minimum tillage is shown in Table 4.14. In the base run, maximum crop acreage under minimum tillage is obtained in lake states and corn belt regions. This remains to be the case in the other energy alternatives also. As the energy situation changes, at national levels larger acreage is brought under minimum tillage as compared to base run. At regional level, differences exist, but the changes resulting in larger acreage under minimum tillage outweigh the changes in opposite directions. Regional differences arise due to a number of factors determining the comparative advantage of selecting a particular method of crop production in a region.

The total nitrogen used by the endogenous crops is summarized at MR and national level in Table 4.15. Commercial fertilizer nitrogen requires natural gas and electricity for manufacture, whereas livestock nitrogen does not. In base run where energy prices are low and supplies are unrestricted all livestock nitrogen supply is exhausted and the livestock nitrogen supplies approximately 25 percent of the nitrogen used by endogenous crops. As energy prices increase or energy supplies decrease, no change in the use of livestock nitrogen therefore are anticipated. Under all four energy alternatives considered livestock nitrogen remains at the same level contributing up to 26 percent total nitrogen used by the endogenous crops in the

Table 4.14. Distribution of crop acreage under minimum tillage, by market region, under alternative energy situations

	Low energy prices	High energy prices	National energy supply restriction	Regional energy supply restriction
	-----millions of acres-----			
Northeast	7.22	7.22	7.22	7.22
Southeast	3.52	3.52	1.99	3.26
Lake states	11.27	11.27	11.27	11.27
Corn belt	17.30	16.76	23.94	25.40
Delta states	3.40	3.40	3.40	3.40
Southern plains	6.16	8.48	11.86	11.86
Mountain	0	5.15	5.15	5.15
North Pacific	0	0.04	0.17	1.60
South Pacific	0.10	0.10	3.87	3.69
U.S. TOTAL	48.97	55.93	68.87	72.84

Table 4.15. Nitrogen use for endogenous crops, by market region, under alternative energy situations

	Low energy prices		High energy prices		National energy supply reduction		Regional energy supply reduction	
	Livestock nitrogen	Commer- cial fertilizer	Livestock nitrogen	Commer- cial fertilizer	Livestock nitrogen	Commer- cial fertilizer	Livestock nitrogen	Commer- cial fertilizer
	-----million tons-----							
Northeast	385.81	751.43	385.81	778.62	385.81	949.34	385.81	1097.14
Southeast	299.46	580.81	299.46	605.81	299.46	384.36	299.46	449.34
Lake states	623.20	1832.97	623.20	1793.91	623.20	1799.21	623.20	1569.42
Corn belt	1389.15	3643.66	1389.15	3659.10	1389.15	3909.77	1389.15	3627.76
Delta states	224.73	1019.07	224.73	1031.05	224.73	958.06	224.73	886.74
Southern plains	362.03	2498.89	362.03	2455.75	362.03	1907.09	362.03	2325.97
Mountain	402.82	600.96	402.82	583.34	402.82	521.31	402.82	519.10
North Pacific	140.34	938.22	140.34	903.42	140.34	900.25	140.34	846.71
South Pacific	202.90	313.21	202.90	318.70	202.90	315.80	202.90	460.37
U.S. TOTAL	4030.44	12,179.22	4030.44	12,129.72	4030.44	11,645.18	4030.44	11,782.81

case of national level energy supply reduction. Use of commercial fertilizer nitrogen reduces in the energy alternatives of high prices and reduced supplies as compared to the base run. Due to changes in cropping patterns, regional differences exist in nitrogen use changes at regional level. For example, with higher energy prices, use of commercial nitrogen decreases in lake states and southern plains. But use of commercial nitrogen increases in corn belt and Delta states when energy prices are increased.

Water use by endogenous crops and water prices (rental prices for ground and surface water, and imputed prices for water) are summarized in Table 4.16. Water use by endogenous crops is obtained as follows. Total exogenous water demand is 35.72 million acre-feet (12). The exogenous water demand is assumed to be satisfied by ground and surface water in the same proportion as the total water use. By subtracting the exogenous demand for ground and surface water from total use of ground and surface water, respectively, the use of ground and surface water by endogenous crops is obtained.

Rental price for surface water increases and rental price for ground water decreases as energy becomes more expensive or as energy shortage develops. The rental price for ground water is higher than for surface water in the

Table 4.16. Total water used for endogenous crops, and imputed shadow prices for water at national level

	Low energy prices	High energy prices	National energy supply reduction	Regional energy supply reduction
<u>Surface water:</u>				
Use (mill acre ft)	35.26	33.11	28.98	33.98
Shadow price (\$/acre ft)	4.37	8.64	13.74	14.92
<u>Ground water:</u>				
Use (mill acre ft)	3.09	2.08	0.72	1.28
Shadow price (\$/acre ft)	13.16	8.17	5.14	4.44
<u>Total water:</u>				
Use (mill acre ft)	38.35	35.19	29.70	35.26
Shadow price (\$/acre ft)	15.17	19.52	25.56	30.51

base run because under the particular pattern in which water is supplied, cost of obtaining an acre-foot of water is less for ground water than for surface water. Use of both surface and ground water decreases with higher energy prices and reduced energy supplies. The shadow price of water which is the rental price plus supply price increases as energy prices increase and also when energy supplies reduce. The above results are consistent with the results related to changes in the use of irrigated land under alternative energy situations. Both irrigated land and water used for endogenous crops decline as energy situation changes as compared to the base run.

#### Farm Production and Commodity Prices

Demand for energy in agriculture is found to be price-inelastic by Dvoskin and Heady (18), and Berry (7). Demand for farm products also is price-inelastic (9). Hence, increases in prices of energy inputs to agriculture would result in small decreases in farm output but larger commodity price increases. Production levels of endogenous crops are summarized in Table 4.17. Commodity prices at farm level are summarized in Table 4.18.

Changes in production levels are limited by the nature of demand structure specified in the model.



Table 4.17. National level production of farm products

	Units	Low energy price scenario	High energy price scenario	Low energy prices national level reduction in energy supply	Low energy prices regional level reduction in energy supply
Feedgrains <sup>a</sup>	Mill. bu	8916.2	8916.2	8888.42	8923.0
Wheat	Mill. bu	2468.9	2468.3	2482.6	2429.9
Soybeans	Mill. bu	2164.5	2085.1	1940.6	1940.6
Cotton	Thous. bales	12000.0	12000.0	12000.0	12000.0
Hay	Mill. tons	134.1	134.1	134.1	134.1
Silage	Mill. tons	117.1	117.1	117.1	117.1

<sup>a</sup>Feedgrain production is expressed in corn equivalents: comprised of corn, barley, oats and sorghum.

Table 4.18. National level prices of farm commodities, at farm level, under alternative energy situations

	Units	Low energy price scenario	High energy price scenario	National level energy supply restriction	Regional level energy supply restriction
Feedgrains	\$/bu	2.41	2.80	3.89	5.07
Wheat	\$/bu	3.80	4.36	6.39	8.68
Soybean	\$/bu	7.16	7.92	10.02	11.93
Cotton	¢/lb	34.98	40.27	48.73	69.27

Production of feedgrains for example, decreases when energy supply is decreased at national level. But wheat production at the same time increases. The changes are compensating, to some extent, due to the constant level of feed demand by exogenous livestock sector for grains. Although domestic demand for feed grains reduces (due to higher prices for feedgrains) for food and industrial uses, and feed demand also reduces, in order to satisfy the remaining feed demand wheat production increases. Within the aggregate commodity "feed grain", production of oats is actually larger when energy supplies are reduced. The increased output of oats is used to meet livestock feed demand. Production of both barley and corn is lower under national level energy supply reduction scenario. Thus, depending upon the energy required to produce an individual grain crop, the least cost combination of feedgrains is obtained to meet the fixed level of feed demand. Therefore, in the case of wheat and feed grains, although certain components of demand decline due to higher commodity prices substitution in demand for livestock feed may actually increase the total production despite higher commodity prices.

But in the case of soybean, as commodity prices increase due to either increased energy prices or reduced energy supplies, output decreases as compared to the base run. When energy supplies are restricted at regional level

total soybean output is at lowest level compared to other three energy situations.

In the case of cotton, hay and silage output levels are held fixed at exogenously specified levels in all four energy situations. Consequently, as energy prices, increase, and supplies reduce, shadow prices of cotton and roughages increase.

Comparing the base run results with the situation in 1978-79, feed output of 8916.2 million bushels is larger than the USDA (68) estimate of total (including exports) U.S. consumption of feed grains at 8405.12 million bushels. The higher export demand, feed demand and larger population base used in deriving the demand functions may account for large part of the difference. Similar differences exist with respect to wheat and soybeans and they can be explained in the same manner as in the case of feed grains.

Prices are more sensitive to changes in the model than in the real world situation and hence do not compare as well as quantities with the real world situation. The current dollar prices received by farmers in 1978-79 were 2.20 dollars per bushel for corn (comparable to feed grain price), 2.94 dollars per bushel for wheat, and 6.75 dollars per bushel for soybean. When converted into current dollars the base run prices for the same commodities (see Table 4.18) are much higher than the 1978-79 prices. But the higher

prices result also due to increased demand from an outward shift in demand as population base increases.

#### Cost of Production, Energy Expenses, Farm Income and Consumer Food Cost

Energy prices and energy supplies are not controlled by the farming sector. As an input in farm production, however, energy influences costs and returns in farming. As often pointed out during the discussion of results in the present study, the low price elasticities of demand - for energy in agricultural production, and for farm products in consumption - magnify the impact of changes in energy situation on farm production cost and consumer expenditures on farm products.

In Table 4.19 national level net farm income, cost of production and consumer expenditures related to crops endogenous to the model are presented. As energy prices increase or supplies are reduced, the net farm income actually increases (see Table 4.19). This result is similar to the result obtained by Dvoskin and Heady (18). However, as energy prices increase and supplies grow shorter, cost of production also increases.

Cost of production is maximum when energy prices increase; and farm incomes are maximum when energy supply is restricted at MR level. Consumer expenditures are also

Table 4.19. National level net farm income from endogenous crops

	Farm income in 1975 \$ <sup>a</sup>	Cost of production <sup>b</sup>	Consumer expenditures <sup>c</sup>
	-----million dollars-----		
Low energy price scenario	16,979.45	32,118.07	49,097.52
High energy price scenario	17,545.58	36,587.45	54,133.03
Low energy prices and national level energy reduction	29,721.54	32,341.36	62,062.9
Low energy prices and regional level energy reduction	37,438.30	33,417.25	70,855.55

<sup>a</sup>Return to land, labor, water and energy.

<sup>b</sup>Variable cost of production.

<sup>c</sup>Sum of farm income and cost of production.

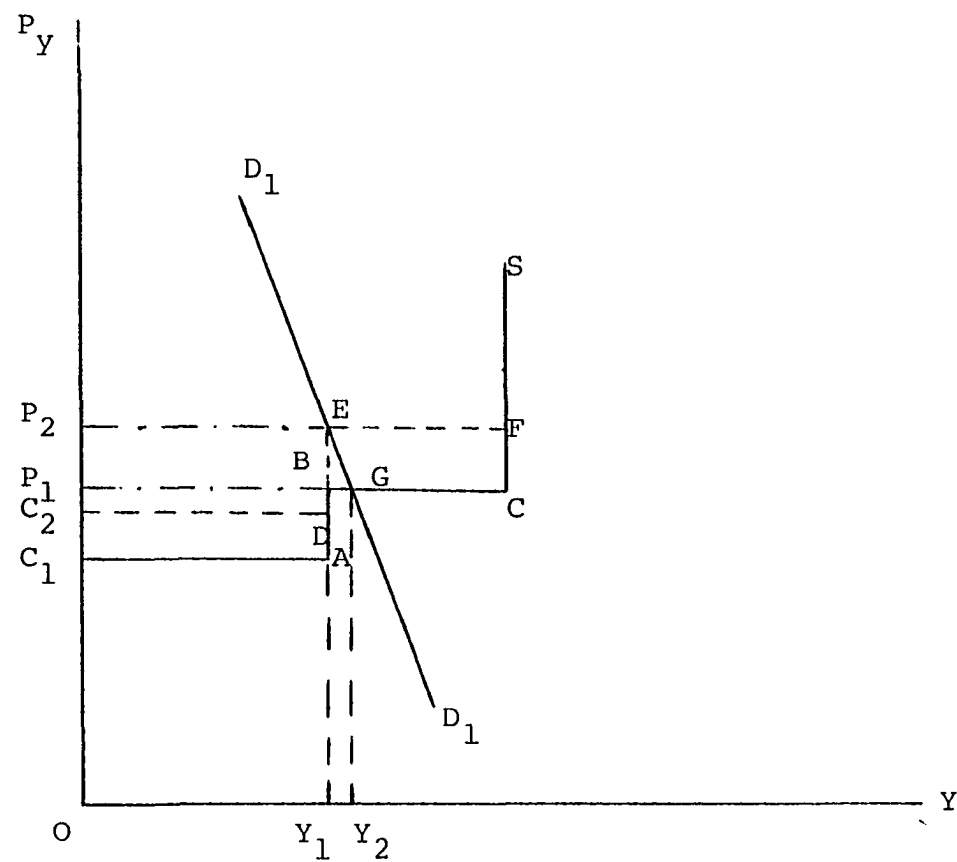


Figure 4.2. Illustration of income and cost

highest when energy supplies are restricted at MR level. The situation may be explained with the help of Figure 4.2.

Let  $D_1D_2$  be the inelastic linear demand curve for commodity  $y$ . Quantity of  $y$  is represented on horizontal axis and price on the vertical axis. Let  $C_1ABCS$  be the initial supply function for  $y$ . Then equilibrium price and output under competition are  $OP_1$  and  $OY_2$ , respectively. Total production cost is given by the area under the supply curve up to  $OY_2$ . Consumer expenditures equal the area of  $P_1GY_2O$ . Thus net income to producers is the difference between consumer expenditure and production cost, given by the area of rectangle  $C_1ABP_1$ . As input prices increase, let the supply curve for  $Y$  shift upward to  $C_2DEFS$ . The new equilibrium price and quantity of  $Y$  are  $OP_2$  ( $<OP_1$ ) and  $OY_1$  ( $<OY_2$ ), respectively. Consumer expenditures are given by the area of rectangle  $OY_1EP_2$ . Production cost is the area of rectangle  $OY_1DC_2$  and producers' income is  $C_2DEP_2$ . Results from price theory suggest that as input prices increase, output supply shifts upward in the case of normal factors. With inelastic demand for output this implies that equilibrium output level will reduce and a greater (in percentage terms) increase in consumer expenditures on the output will result. But direction of change in income is dependent on the magnitude and nature of shift in



the supply function.

Reduction in the supply of a particular input used in production also has the effect of shifting the supply curve upward (as more expensive inputs and methods of production are brought in by producers to replace the scarce input). Thus again, consumer expenditures increase and direction of income change is dependent on the nature and magnitude of shift in the output supply function.

A breakdown of the cost of farm production obtained under alternative energy situations at national level is shown in Table 4.20. Under higher energy prices, increases in energy input prices outweigh the decrease in nonenergy cost of farm production. The reduction in nonenergy expenses is a result of changes such as reduced use of nitrogen, transportation and also irrigation. When energy supplies are restricted, whether at national or regional level, energy expenses are smaller than when energy supplies are unrestricted. However, now production takes place by substituting other inputs which are more expensive, for energy and consequently total production cost increases.

It may be argued that restriction on the supply of energy at national level may be more "efficient" as compared to restriction of energy supplies are restricted at regional level, as farm production takes place with more

Table 4.20. Cost estimates for endogenous crop production, at national level, under alternative energy situations

	Low energy prices	High energy prices	National energy supply reduction	Regional energy supply reduction
	-----millions of dollars-----			
<u>Crop production</u>				
Energy cost	4481.76	8658.31	4128.31	3984.34
Nonenergy cost	21798.05	21946.65	23216.13	23586.71
<u>Nitrogen use</u>				
Energy cost	1398.33	1895.49	1354.18	1376.98
Nonenergy cost	2639.90	2356.43	1863.98	2674.56
<u>Irrigation</u>				
Energy cost	135.94	126.30	91.35	104.20
Nonenergy cost	182.39	193.85	188.64	203.84
<u>Transportation</u>				
Energy cost	30.05	38.46	12.00	14.96
Nonenergy cost	1451.65	1371.96	1486.77	1471.66
TOTAL ENERGY COST	6046.08	10718.56	5585.84	5480.48
TOTAL NONENERGY COST	26071.99	25868.89	26755.52	27936.77
TOTAL COST	32118.07	36587.45	32341.36	33417.25

expensive and less productive inputs and methods of farming. The opportunity for concentration of production in the most productive and least expensive regions which exists under a national level energy supply restriction is removed when energy is restricted at regional level. While production may be "inefficient", as previously noted, farm incomes are highest under MR level energy supply restriction. On the other hand, consumer expenditures on farm products considered are also maximum when farm incomes are maximum.

## CHAPTER V. SUMMARY, LIMITATIONS AND IMPLICATIONS

Developments in the energy sector in recent years have suggested that in the future, energy will continue to be a very expensive factor of production in all its uses including agriculture. In the United States, agriculture is a highly mechanized and energy-intensive industry. Production expenditures related to energy constitute an important portion of total farm production costs. As alternatives to energy-intensive systems may not yet be economical to apply, the impact of rising energy prices would be for the production costs to increase substantially in farm production. Energy price increases affect agriculture in indirect ways also. The impact of increasing energy prices on inflation would affect demand for farm products and prices of other inputs. Agriculture as a source of energy is also becoming an increasingly practical concept. The present study measures the impact of rising energy prices and reduced energy supplies on resource use, production patterns, farm product prices, production costs, farm incomes and consumer expenditures on farm products.

A quadratic programming model incorporating linear, interdependent demand functions for corn, barley, oats, wheat and soybean oil for U.S. agriculture is formulated.

The continental United States is divided into 105 agronomically homogeneous producing areas and the producing areas are aggregated into nine market regions. Crop production is represented by a cost-minimizing linear program sub-model. Crop rotations are defined at producing area level.

The quadratic programming model yields optimal values for production of output and resource use consistent with the competitive market structure. The quadratic programming model is converted into a separable programming model. Transformation of variables needed for obtaining separability in the objective function is achieved by using eigenvalue transformations.

The model simulates supply and demand for feed grains, wheat, soybean and roughages in the output markets; and land, water, nitrogen and energy in the input markets. Crop production is defined on dry land and irrigated land. Three ways of crop production specified in the model are - conventional tillage with residue left, conventional tillage with residue removed and minimum tillage. The per acre requirements of nitrogen, phosphorous and potassium fertilizers, water for irrigated crops and energy inputs in crop production, crop drying, transportation, irrigation and manufacture of fertilizers and pesticides is specified exogenously to the model. Crop yields are also predetermined to the model. Energy requirements are specified in terms of

diesel (gasoline requirements are specified in terms of diesel fuel equivalents), natural gas, electricity and liquid petroleum gas (LPG). Four solutions of the mathematical programming model are obtained each simulating the impact of a given energy situation on the crop sector of U.S. agriculture. The four alternative situations are: 1) low energy price scenario based on anticipation of relatively small energy price increases; 2) high energy price scenario based on anticipation of high energy prices; 3) reduced energy supplies with low energy prices - a 10 percent reduction in energy supplies from the level of energy use under low energy price scenario is imposed at national level; and 4) energy supplies are reduced by 10 percent of energy used under low energy price scenario, but the energy supply restrictions are imposed at market region level.

The impact of alternative energy situations on agriculture is evaluated in terms of a number of variables such as crop production, resource use, input and output prices, farm income and food costs. The low energy price scenario is considered as the base run in the study and changes from the base run in the remaining three alternatives are evaluated. The base run results may be interpreted as reasonable approximations of U.S. agriculture in terms of production levels and acreages. Prices tend to be higher than the

prevailing levels and accordingly, net farm income and consumer expenditures are also higher in the base run than the prevailing levels.

As energy prices increase, use of energy decreases. Measured in terms of megacalories, under high energy price scenario, total energy use declines by 2.6 percent for about 41.05 percent increase in energy price, as compared to the level of energy use in the base run. This result confirms the results of the studies conducted by other researchers that demand for energy in agriculture is price-inelastic. But several adjustments take place in order to reallocate resources and enterprises so that an efficient organization of crop production sector under altered energy situation is achieved.

Table 5.1 summarizes the changes in input use at national level under alternative energy situations. As energy prices increase, maximum impact among energy inputs is on the use of LPG. A decrease of 7.7 percent in the use of LPG is observed as compared to the base run. Use of other energy inputs also is reduced as they become more expensive relative to other inputs. Use of all inputs is reduced with increased energy prices. But inputs which use energy either in their manufacture or crop use, are affected more than others. For example, use of water decreases

Table 5.1. National level use of inputs in agriculture<sup>a,b</sup>

	Units	Low energy price scenario	Low energy price scenario	Low energy prices and national level energy supply reduction	High energy prices and regional level energy supply reduction
<u>Energy</u>					
Diesel	Mill. gal	4570.58	4469.65 (-2.2)	4273.55 (-6.5)	4281.05 (-6.3)
Electricity	Mill. kwh	7686.85	7346.57 (-4.4)	6451.30 (-16.1)	6632.87 (-13.71)
Natural gas	Bill. cu ft	357.99	354.07 (-1.1)	338.78 (-5.4)	342.59 (-4.3)
LPG	Mill. gal	588.48	543.49 (-7.7)	556.70 (-5.4)	502.86 (-14.6)
TOTAL ENERGY	Bill. mcal	264.05	257.08 (-2.63)	237.65 (-10.0)	237.65 (-10.0)
<u>Nitrogen</u>					
Commercial fert.	Thous. tons	6739.27	6714.50 (-0.4)	6472.25 (-4.0)	6540.86 (-2.9)
Livestock	Thous. tons	2015.22	2015.22 (0.0)	2015.22 (0.0)	2015.22 (0.0)
TOTAL NITROGEN	Thous. tons	8754.49	8729.72 (-0.3)	8487.47 (-3.1)	8556.08 (-2.3)
<u>Land</u>					
Dry land	Mill. acres	336.51	335.36 (-0.3)	330.37 (-1.8)	334.79 (-0.5)
Irrigated land	Mill. acres	20.69	19.39 (-6.3)	17.20 (-16.9)	18.70 (-9.6)
TOTAL LAND	Mill. acres	356.20	354.75 (-0.4)	347.57 (-2.4)	353.49 (-0.8)
<u>Water</u>					
Surface water	Mill. acre-ft	67.52	65.69 (-2.7)	63.27 (-6.3)	68.63 (+1.6)
Ground water	Mill. acre-ft	5.95	4.22 (-29.1)	2.15 (-63.9)	2.35 (-60.5)
TOTAL WATER	Mill. acre-ft	73.47	70.91 (-3.5)	65.42 (-11.0)	70.98 (-3.40)

<sup>a</sup>The energy, nitrogen and water use shown in this table includes the inputs used by exogenous crops.

<sup>b</sup>Numbers in parentheses are percentage change over low energy price scenario.



by 3.5 percent under high energy price scenario as compared to water used in the base run. But use of land is decreased by only 0.4 percent. As another example, ground water which requires higher amounts of energy than surface water decreases by 29.1 percent whereas use of surface water declines by only 2.7 percent. Similar results are observed in the case of irrigated and dry land use and nitrogen from commercial fertilizer and livestock wastes, used in crop production.

With energy shortage or conservation imposed at national level, pattern of reduction in energy inputs is slightly different from the pattern observed in the case of increased energy prices. Electricity use is reduced more than any other energy input. Reduction in the use of land, nitrogen and water is greater with energy supply reductions than in the case of higher energy prices.

The regional level energy supply reductions suggest a different pattern in resource adjustments than national level energy supply reduction. Among energy inputs, LPG's use is reduced (by 14.6 percent) more than any other energy input compared to base run. Reductions in use of land, water and nitrogen are less under the regional level energy shortage than under the national level energy shortage. Use of surface water actually increases as energy supply reduced at regional level as compared to the use of surface

water is the base run. However, total water used is less under restricted energy supplies as compared to the water use in the base run.

Reductions in input use occur due to two factors. They are: a) substitution of less expensive and less scarce inputs among those available for more expensive and scarce inputs to produce a given level of output; and b) change in the crop-mix and reduction in total crop output. The overall reductions in input use observed previously may be attributed to the drop in total crop production caused by increased energy prices and reduced supply of energy. Changes in input mix, such as greater use of dry land as compared to irrigated land, greater use of surface water relative to use of ground water may be attributed to the substitution among inputs due to changes in relative prices and also due to change in crop-mix. While total feed grain production remains unchanged from its base run level when energy prices increase, composition of feed grains does change. A greater proportion of livestock feed grain demand is satisfied by oats and sorghum reducing quantities of barley, corn and wheat used as livestock feed. Production of soybeans is reduced by 3.7 percent as compared to base run, when energy prices increase. Production of wheat declines only slightly. Low price elasticities of demand grain crops cause only small reductions

in quantities demanded despite increases in product prices. Feed grain prices increase by 16 percent, wheat price increases by 15 percent and soybean price increases by 11 percent. Changes in output levels are more significant when energy supplies are reduced rather than when energy prices rise. National level energy supply reduction results in reduced feed grain output but increased wheat production. A change in the composition of livestock feed demand causes the partially compensating changes in the production of feed grains and wheat. Soybean output reduces by 10.3 percent as compared to the base run under national level energy supply reduction. In all cases of altered energy conditions, as compared to the base run, prices of crops produced increase.

Imputed input prices also increase when energy prices increase and supplies grow shorter. Under national level energy supply restriction, a shadow price of 8 cents per megacalorie of energy was obtained. Under regional energy supply reduction, the shadow price of energy increases by about 6 cents per megacalorie over the shadow price under national level energy cut. The high shadow price of energy obtained in the study reflects the value of additional megacalorie of energy supplied to agriculture.

Increases in energy prices result in higher production expenses and also higher net farm incomes. While nonenergy

costs in farm production decrease, costs related to energy use increase significantly. Net farm income from endogenous crops in the base run was estimated to be 16.98 billion dollars. When energy prices increase, the net income rises to 17.55 billion dollars. In 1978-79 total (from crop and livestock sources) net farm income was estimated to be 19.8 billion dollars (68). With reductions in energy supplies at national and regional levels, net farm income from endogenous crops increases by 75 percent and 121 percent, respectively over the base run. As energy supply reduces, nonenergy expenses increase while the energy-related expenses decrease; production takes place on lower yield producing areas; and low-yield methods of production requiring less energy are adopted on a wider scale. Thus, the nonenergy cost of production per unit of output increases.

The total cash receipts of the farm sector, and hence consumer expenditures on food items increase under higher energy prices and lower energy supplies as compared to the base run energy situation.

#### Limitations of the Study

The nature and extent of adjustments in agriculture which can be obtained from the present study are preordained in the model used. A "better" model will yield better

results. A major area which needs improvement over the present status is the demand structure specified in the model. The linear demand functions used in the study are derived from price elasticity estimates by Brandow (9). Use of more recent data should improve the performance of the model.

Livestock sector is specified to be exogenous in the present study. Livestock sector is a major source of demand for feed grains and roughages. Hence, changes in prices of crops which are inputs to livestock sector affect livestock production and feed demand. This aspect of impact of changes in energy situation could not be considered in the present study.

Provision of greater opportunity for adjustment in resource use and farming methods may suggest different pattern of adjustments in agriculture. For example, if conversion of irrigation equipment from dependence on natural gas or LPG to diesel as fuel source were to be permitted, reductions observed in water use in farming may have been smaller. Similarly, if lower nitrogen rates for crops were permitted different extent of resource adjustment may be realized.

The study does not deal with the indirect effects of energy situations on agriculture. Policy choices in the energy sector have effect on the general state of the economy.

These effects in turn influence farming sector. Similarly, impact of changing energy situation on other components of the food system which are as much energy intensive as the farm production sector are not considered in this study. Effect of changing energy situation on the potential role of agriculture as an energy producer is also a relevant area which is not considered in this study.

Separable programming used in the study does not provide exact results. Therefore, the optimality conditions which are identified with competitive equilibrium are only approximately satisfied. A more crucial limitation in using separable programming is the sensitivity of the results to changes in segments used to linearize the nonlinear expression. Thus although separable programming can be used advantageously, its limitations should be noted.

Finally, the present study employs a "normative" model. The solutions or results of analysis are those which are consistent with the theoretical conditions of partial competitive equilibrium. As a result, the extent of changes suggested by the results of the study are mainly potential and suggest the long-term trends.

### Implications of the Study

The enormous importance of energy from fossil fuel sources to agriculture or the food system in the United States has been adequately discussed by researchers (54). Lockeretz (37) brings together several studies on relationships between energy and agriculture.

Economic analyses of interrelationships between energy inputs and agriculture have suggested that demand for energy in agriculture is price-inelastic (18). Results from the present study also support this view. The inelastic nature of demand for energy with respect to price implies that farm production expenses will rise as energy prices increase. Further, to achieve reductions in energy use in farming, direct reductions in energy supply may be more reliable than increases in energy prices.

One important cause of low price elasticity of demand for energy apart from the technology itself is the low price elasticity of demand for farm products. Although prices of farm products may increase due to increased production cost, demand for farm output does not reduce significantly. As a result, consumer expenditures on farm products increase and farm incomes also may increase. But these results are obtained under the optimal allocation of resources and consumer and producer behavior as postulated in

consumer and firm theories. Also, the results relate to the situation when final adjustments are made and only the specified changes in the situation.

Increased use of heavy machinery and agricultural chemicals in farming has been running counter to concern among the public and policy-makers with respect to soil conservation and preserving a clean environment. As results of the present study indicate, as energy situation develops into one characterized by higher energy prices or reduced energy supplies, crop acreage under minimum tillage will increase and application rates of fertilizers may decrease. Rate of pesticide application, however, may increase. Thus, certain developments due to energy crisis in agriculture may help achieve certain environmental goals of the society.

Energy crisis does not affect all regions uniformly. The present study suggests reductions in irrigated farming as a major adjustment in agricultural sector in response to increases in energy prices or reduced supplies of energy. However, recent data on irrigated farming do not support this finding. Our findings are however based on increases in energy prices far greater than the prevailing energy prices. Also, results of this study should be considered as indicators of long-term change under prescribed conditions.



Farm income support programs in the United States have mainly evolved around price support and supply control programs. Under supply control, land retirement has been the major policy instrument. Thus, increasing farm incomes by reducing the use of one of the essential inputs in production is not a new result. In this light, increases in farm income due to reductions in energy use may be understood more clearly. The results however, suggest an increased use of land and reduced use of energy as the relevant measure for increasing farm income, under changed energy.

The study analyzed some aspects of impact of restricting energy supplies at national and regional levels. As may be expected, production costs are higher when energy supplies are restricted at regional level rather than at national level. Restricting energy supplies at regional level prevents efficient interregional allocation of resources. Thus, from an efficiency standpoint (as suggested by the increased cost of production), regional level energy supply reduction is less preferred to the national level energy supply reduction alternative. But increased farm incomes and reduced costs of adjustment (costs are greater under national level energy cut) to rural communities make regional level energy supply reduction more favorable.

Results of the present study also imply that changes

may occur in the composition of feed grains fed to livestock. Although the nature of trade-off between feeding grain and roughages to livestock is not clear, importance of grains other than corn may increase as corn is relatively more energy-intensive crop.

United States has been the "bread-basket" of the world for many years now. Implications of increased food production cost in United States are manifold to the rest of the world. Agricultural production technology of United States, as in other industrialized nations, is characterized by relatively large input of energy. Although this technology is not exactly duplicated around developing nations of the world, some of the basic principles such as "adequate fertilization of soil" and "increased irrigation" are their major source of increased food production. As both fertilizers and irrigation are energy intensive inputs, technology based on their use becomes increasingly expensive when energy prices increase. Thus, increased food production in less developed countries may become less uncertain and the role of the United States as food producer will grow. But, increased energy expenses will make U.S. food more expensive. If the less developed countries can find means to finance food imports from the United States and the more affluent countries continue to import food from the United States, agriculture may become a major foreign

exchange earner, as the current situation shows, in future  
for the nation.

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APPENDIX: DIAGONALIZATION OF THE DEMAND MATRIX  
USING EIGENVALUES AND EIGENVECTORS

The quadratic programming problem of competitive equilibrium (problem III in Chapter II) is rewritten as follows.

Problem IIIa:

$$\text{Maximize } z_{3a} = d'_0 q + \frac{1}{2} q' D q - c' x \quad (\text{A.1})$$

$$\text{such that } Bx \leq 0; \quad (\text{A.2})$$

$$q - Ax \leq 0; \quad (\text{A.3})$$

and

$$q, x \geq 0 \quad (\text{A.4})$$

where  $D$  is a square, real, symmetric and negative semi-definite matrix of size  $n$ ;  $Dq + d = p$  is a set of inverse linear demand functions; and other expressions are as defined in Chapter II of the main text.

A result on the real, symmetric and square matrices can be stated as follows (5): "Every real symmetric matrix  $A$  is orthogonally similar to a diagonal matrix whose diagonal elements are the characteristic roots of  $A$ ."

i.e., we can write,

$$A = TVT' \quad (\text{A.5})$$

where,

$V = (n \times n)$  the diagonal matrix with the characteristic roots (eigenvalues) of  $D$  as its diagonal elements; and

$T = (n \times n)$  orthogonal matrix such that  $T'T = I_n$ , an identity matrix.

Moreover, matrix of eigenvectors of  $D$  can be identified as the matrix  $T$ (32).

Then, we write the objective function, Equation (A.1), of problem IIIa as follows.

$$z_{3b} = d_0'q + \frac{1}{2}q'TVT'q - c'x \quad (A.6)$$

Define a vector of new variables ( $z_i$  with  $i = 1, 2, \dots, n$ ) such that,

$$z = T'q \quad (A.7)$$

where  $z$  is an  $(n \times 1)$  vector.

Now the competitive equilibrium problem can be rewritten as:

Problem III:

$$\text{Maximize } z_{3b} = d_0'q + \frac{1}{2}a'Vz - c'x \quad (A.8)$$

such that

$$Bx \leq b \quad (A.9)$$

$$q - Ax \leq 0 \quad (A.10)$$

$$T'q - z = 0 \quad (A.11)$$

$q, x \geq 0$  and

$z$  is unrestricted in sign (A.12)

The constraints in the Lagrangian constraint set are those from (A.9) to (A.12) and,

$$Vz - I\lambda_3 = 0 \quad (A.13)$$

$$-I\lambda_1 + T\lambda_3 \leq -d \quad (A.14)$$

$$A'\lambda_1 - B'\lambda_2 \leq c \quad (A.15)$$

$$\text{with } \lambda_1 \text{ and } \lambda_2 \geq 0; \text{ and} \quad (A.16)$$

$\lambda_3$  unrestricted in sign.

Consider the constraint (A.14)

$$-I\lambda_1 + T\lambda_3 \leq -d.$$

Substituting from constraint (A.12),

$$-I\lambda_1 + TVz \leq -d$$

or

$$\lambda_1 \geq TVz + d \quad (A.16)$$

From (A.11) we have,

$$z = T'q.$$

Hence,

$$\lambda_1 \geq TVT'q + d \quad (A.17)$$

As  $TVT' = D$  we can write,

$$\lambda_1 \geq Dq + d \quad (A.18)$$

From Kuhn-Tucker conditions (52) we know that for optimal



values of  $\lambda_i$  and  $q$ ,

$$\lambda_1^* = Dq^* + d, \quad \text{or} \quad q^* = 0. \quad (\text{A.19})$$

Therefore,  $\lambda_1^* = p$ , and remaining constraints ensure that for optimal values of  $q$ ,  $x$ ,  $z$ ,  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ , transformation of variables in the objective function leaves the problem consistent with the competitive equilibrium conditions.

The national demand functions were converted to regional level using regional population proportions, in this study. The disaggregation of demand matrix involved in the procedure is accomplished as follows:

Let  $\bar{D}$  be the national level linear demand matrix. Then demand matrix for  $k^{\text{th}}$  region is obtained as,

$$\bar{D}^k = a_k \cdot \bar{D} \quad (\text{A.20})$$

where

$\bar{D}^k$  = demand matrix for  $k^{\text{th}}$  region;

$\bar{D}$  = national level demand matrix; and

$a_k$  = projected proportion of population in the  $k^{\text{th}}$  region.

In terms of eigenvalue transformations,

$$D^k = a_k^{-1} \cdot (\bar{D})^{-1} \quad (\text{A.21})$$

$$= a_k^{-1} \cdot D \quad (\text{A.22})$$

$$= a_k^{-1} \text{TVT}' \quad (\text{A.23})$$

where

$D$  = the matrix of inverse demand functions;

$V$  = the diagonal matrix of eigenvalues of  $D$ ; and

$T$  = matrix of eigenvectors of  $D$ ,

or

$$D^k = T \bar{V}_k T' \quad (A.24)$$

where

$$\bar{V}_k = a_k^{-1} \cdot V \quad (A.25)$$

Therefore, only eigenvalues need to be transformed in disaggregating national level demand matrix to regional level. The matrix of eigenvectors remains unchanged.

The matrix of national level inverse demand functions used in the study is,

$$D = - \begin{bmatrix} 0.0002 & 1.7581 \times 10^{-7} & 2.7755 \times 10^{-7} & 3.4971 \times 10^{-7} & 0.4390 \times 10^{-7} \\ & 0.0001 & 0.0001 \times 10^{-1} & 0.0001 \times 10^{-1} & 0.8241 \times 10^{-7} \\ & & 0.0028 & 0.0002 \times 10^{-1} & 2.5785 \times 10^{-7} \\ \text{symmetric} & & & 0.0002 & 3.2288 \times 10^{-7} \\ & & & & 0.0002 \end{bmatrix}$$

The matrix of eigenvectors is,

$$T = \begin{bmatrix} -0.0023 & -0.2041 & -0.9788 & 0.0149 & -0.0001 \\ 0.9777 & -0.0022 & 0.0013 & 0.2101 & -0.0042 \\ -0.0022 & 0.00002 & -0.00004 & -0.0097 & -0.9999 \\ -0.2101 & -0.0094 & 0.0173 & 0.9774 & -0.0091 \\ -0.0003 & 0.9789 & -0.2039 & 0.0130 & -0.0001 \end{bmatrix}$$

And the regional level diagonal matrices of eigenvalues are obtained as submatrices of,

$$M = \begin{bmatrix} 0.3364 \\ 0.1169 \\ 0.1345 \\ 0.1056 \\ 0.0510 \\ 0.0863 \\ 0.0357 \\ 0.0289 \\ 0.1047 \end{bmatrix} \otimes \begin{bmatrix} -0.0001 & 0 & 0 & 0 & 0 \\ & -0.0002 & 0 & 0 & 0 \\ 0 & 0 & -0.0002 & 0 & 0 \\ 0 & 0 & 0 & -0.0002 & 0 \\ 0 & 0 & 0 & 0 & -0.0028 \end{bmatrix}$$

Where  $\otimes$  implies Kroenecker product. Note that the values reported above have been rounded off to the nearest decimal place.

The matrix of eigenvalues for  $k^{\text{th}}$  region is the sub-matrix of M given by rows from k to k+5, and columns from k to k+5.